State-space representation—Introduction

State-Space Modelling of Systems with no Input Derivatives

State-Space Modelling of Systems with Input Derivatives

How to solve state equation?

Systems and Simulations—Lecture 5

State-Space Modelling Approach

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How to solve state equation?

State-space representation—Introduction

- Modelling of multiple input multiple output.
- Ease of simulation on computer.
- Powerful approach can be applied in linear and nonlinear systems and to time-varying and time-invariant systems.

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State-space representation—Definitions

State Variables The smallest set of variables $\{x_1, \dots, x_n\}$ that suffice to characterize the behaviour of a dynamic system after a given point in time.

State Vector The vector $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix}$.

State Space The n-dimensional space spanned by $\mathbf{x}(t)$.

State-Space Equations Three types of variables: input, output and state.

Note: State space representation nonunique.

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State-Space Equations

- Consider a system with
  - r input variables $z_1(t), \ldots, z_r(t)$;
  - n state variables $x_1(t), \ldots, x_n(t)$; and
  - m output variables $y_1(t), \ldots, y_m(t)$.
- Variables are related by standard form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} \quad (1)$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{z}. \quad (2)$$

- **A**: State matrix
- **B**: Input matrix
- **C**: Output matrix
- **D**: Direct transmission matrix

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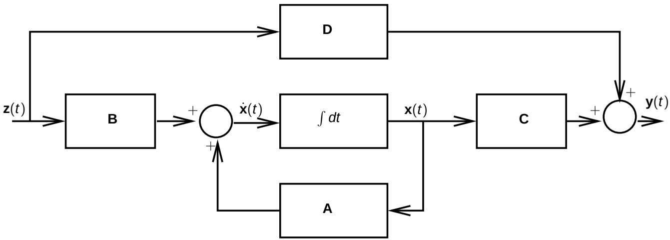
Block Diagram for State-Spce Representation



Example

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Transfer Matrix

- Matlab command for state-space: ss(A,B,C,D)
- Transfer matrix $\mathbf{G}(s)$

$$\mathbf{Y}(s) = \mathbf{G}(s)\mathbf{Z}(s)$$

- Derive an expression for $\mathbf{G}(s)$ in terms of **A**, **B**, **C**, **D**.
- Example

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How to solve state equation?

Case of no Input Derivatives

- Use vairables specifying initial conditions as state variables.
- Example with one mass.
- Example with two masses.

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How to solve state equation?

Case with Input Derivatives

- Cannot use vairables specifying initial conditions as state variables.
- Use system transfer function to obtain auxiliary diffierential equations.
- Use variables specifying initial conditions of the auxiliary equations as state variables.
- Eleminate derivatives to obtain output.
- Example cart with mass damper and spring.

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How to solve state equation?

How to solve state equation?—Homogeneous equations

- Time domain approach
  - Scalar case.
  - Vector case—Matrix exponential.
- Laplace transform approach
  - Scalar and Vector cases.
  - Binomial expansion of $(s\boldsymbol{I} - \boldsymbol{A})^{-1}$.
  - Inverse transform.
  - State transisiton matrix.
- Example

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How to solve state equation?

How to solve state equation?—Nonhomogeneous equations

- Time domain approach
  - Scalar case.
  - Vector case—Matrix exponential.
- Laplace transform approach
  - Scalar and Vector cases.
- Example