CARLETON UNIVERSITY
Department of Systems and Computer Engineering

SYSC 4600 – Digital Communications – Fall 2008
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Full mark: 110 points – closed-book, one-page aid-sheet allowed – 80 minutes

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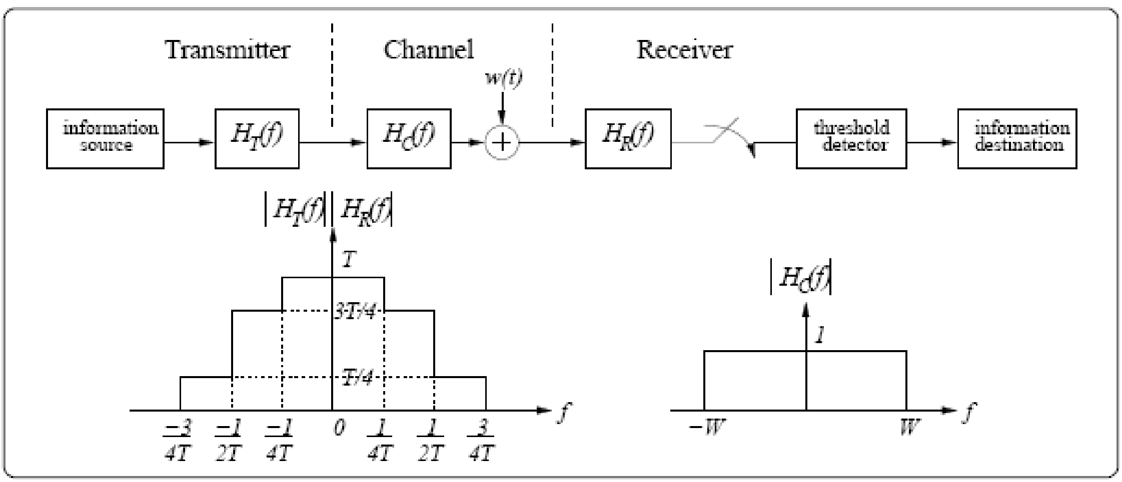
Question 1 (45 pts) **Transmission Through A Bandlimited Channel**

A baseband 16-ary communication system with zero-mean AWGN is shown in the below figure where T denotes the symbol duration. The channel is an ideal LPF with bandwidth W Hz. The transmitter and receiver filters, $h_T(t)$ and $h_R(t)$, constitute a matched filter pair; i.e., $|H_T(f)| = |H_R(f)|$. In the below figure the product $|H_T(f)||H_R(f)|$ is shown as well.



(a) (5 pts) Find the maximum symbol rate R_s, in term of W, for which the channel will not cut the transmitted signal.

(b) (15 pts) At this maximum symbol rate, does the overall system introduce ISI? Justify your answer clearly with the help of a sketch.

(c) (10 pts) If $W = 3$ MHz, what is the maximum bit rate, R_b, of this system?

(d) (10 pts) If the bit rate has to be at least 23 Mbps, what constellation size has to be used instead of 16-ary? In other words, what would be M in the corresponding M-ary case?

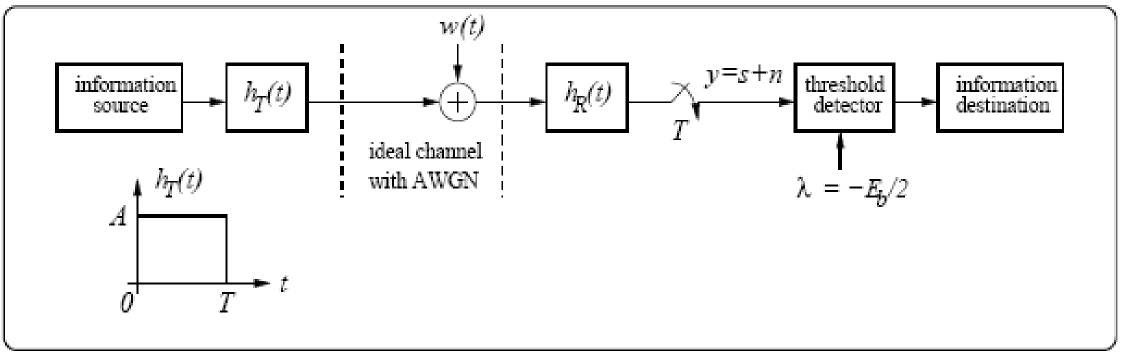
(e) (5 pts) Consider another LPF channel with the same W Hz bandwidth, but with non-ideal passband characteristics (i.e., channel causes spectral shaping). Given that $h_T(t)$ and $h_R(t)$ are the same as above, what operation has to be done at the receiver to remove the ill-effects caused by this channel. Discuss qualitatively.

Question 2 (65 pts) **Binary Signalling with Non-Equally-Likely Bits**

A baseband binary antipodal NRZ signalling scheme is used in a zero-mean AWGN channel (with a PSD of $N_0/2$) as shown in the below figure. The receiver filter is matched to the transmitter filter; i.e., $h_R(t) = h_T(T-t)$.

The source generates independent bits, but with uneven probabilities; $p_1 = 0.8$ and $p_0 = 0.2$. It is given that this situation dictates the use of a treshold level of $\lambda = -E_b/2$ at the threshold detector in order to minimize P_e, where $E_b = A^2T$ is the bit energy. Note that if the bits were equally-likely, λ would have been equal to zero.

At the receiver, the decision variable at the output of the sampler is $y = s + n$, where s and n denote the signal and noise components, respectively.



(a) (10 pts) Find $s|0$, $s|1$, and n.

(b) (15 pts) Find the variance of y in terms of E_b and N_0. Find $f_Y(y|0)$ and $f_Y(y|1)$ and sketch them together; also indicate λ.

(c) (30 pts) Find the probability of error, P_e, in term of the erfc(.) function given below with its argument in terms of E_b and N_0:

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2} du.$$

[Note: this part can be solved by inspection, if you think carefully. If you choose to do so, then state your reasoning very clearly.]

(d) (10 pts) Let $P_{e,I}$ denote the probability of error found in part (c). How does $P_{e,I}$ compare with the probability of error corresponding to a source with equally-likely bits (with $\lambda = 0$), which is known to be $P_{e,II} = \frac{1}{2}\mathrm{erfc}(\sqrt{\frac{E_b}{N_0}})$? In other words, which one of the followings is true:

i) $P_{e,I} > P_{e,II}$, or

ii) $P_{e,I} = P_{e,II}$, or

iii) $P_{e,I} < P_{e,II}$?

Justify your answer; the correct answer without proper justification will not receive any marks.

[Note: you may solve this part based on intuitive reasoning without solving part c).]