Carleton University

Department of Systems and Computer Engineering

SYSC 4600 – Digital Communications – Fall 2009

Assignment #3

Posted on Wednesday, November 4th, 2009

Due on Friday, November 13th at 4 pm in the assignment box.

Marking Scheme: Q1[25%], Q2[25%], Q3[50% + 5% bonus for constellation (e)].

Question 1

A communication system uses two signals $s_1(t) = a\phi(t)$ and $s_2(t) = (a+1)\phi(t)$ to transmit binary one and zero bits respectively, each equally likely, where a is a real number, and $\phi(t)$ is a bandpass signal with energy 1. The communication is corrupted by AWGN with PSD $N_0/2$ and received via a coherent recever with filter matched to $\phi(t)$.

1. Assuming ASK transmission:
   (a) Find a.
   (b) What is the average energy per bit $\mathcal{E}_\mathrm{b}$?
   (c) What is the probability of bit error P_e as a function of $\mathcal{E}_\mathrm{b}/N_0$? You may use formulas from class.
2. Answer the same questions for PSK transmission.
3. Now for any a:
   (a) Find P_e as a function of $\mathcal{E}_\mathrm{b}/N_0$ and a.
   (b) What is the choice of a that gives the smallest P_e for a given $\mathcal{E}_\mathrm{b}/N_0$? (Proof required)

Question 2

Consider a 4FSK system which uses four frequencies $f_1 = 1$ GHz, $f_2 = 1.02$ GHz, $f_3 = 1.05$ GHz, $f_4 = 1.09$ GHz. The four symbols are therefore $s_i(t) = A\cos(2\pi f_i t)$, $0 \le t \le T_\mathrm{S}$. Assume $A = 1$.

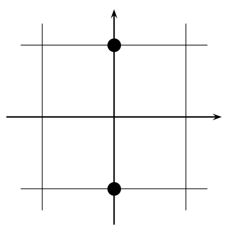
1. Assuming $T_\mathrm{S} = 25$ ns, are the four signals orthogonal?

2. Find the lowest T_S that ensures all four signals are orthogonal. What is the corresponding transmission bit rate?

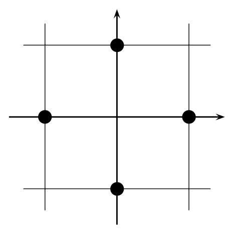
Hint: For both questions, it is best to find $\langle s_i, s_j \rangle$ for general f_i, f_j.

Question 3

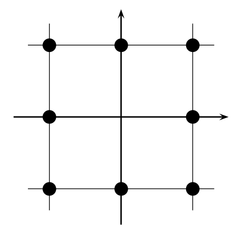
Consider the 6 following signal constellations, with each division representing one unit:



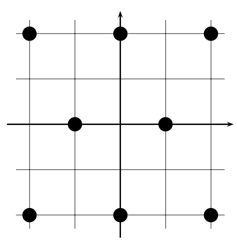
(a) BPSK



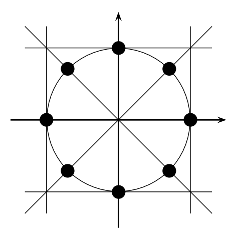
(b) QPSK



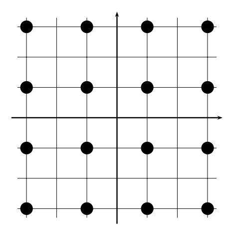
(c) 8QAM



(d) 8QAM–Mod



(e) 8PSK (bonus)



(f) 16QAM

(Note: you may get full marks if you ignore constellation (e), which is worth bonus marks)

1. For each of the 5(6) constellations, find:

   (a) The average energy per symbol $\mathcal{E}_S$ and the average energy per bit $\mathcal{E}_b$.

   (b) Assuming that the probability of error is dominated by errors between the nearest points in the constellation, find $d_{\min}$ the smallest distance between two points. Then find the probability that a given symbol is received with error, using the approximation

$$P_e(\text{symbol}) \approx \tfrac{1}{2}\,\text{erfc}\left(\frac{d_{\min}}{2\sqrt{N_0}}\right)$$

to find $P_{\mathrm{e}}(\text{symbol})$ in the form $\frac{1}{2}\operatorname{erfc}\left(\alpha\sqrt{\frac{\mathcal{E}_{\mathrm{b}}}{N_0}}\right)$ (basically, you must find the constant α for every constellation).

2. If we want to achieve a particular $P_{\mathrm{e}}(\text{symbol})$, sort the constellations from most energy-efficient to least. Justify your answer.

3. Now assume that the symbols are grey–coded, meaning that (practically) every symbol error results in one bit error only. For each of the 5(6) constellations:

   (a) Find the (approximate) probability $P_{\mathrm{e}}(\text{bit})$ that a given received bit is in error, as a function of $\mathcal{E}_{\mathrm{b}}/N_0$.

   (b) Assuming $\mathcal{E}_{\mathrm{b}}/N_0 = 13\,\mathrm{dB}$, find the numeric value of $P_{\mathrm{e}}(\text{bit})$ (hint: use the *MATLAB* function *erfc*).

4. When $\mathcal{E}_{\mathrm{b}}/N_0 = 13\,\mathrm{dB}$, sort the constellations from lowest $P_{\mathrm{e}}(\text{bit})$ to highest.