Lecture 23 Dec. 2, 2015

- Baseband: w/o modulation
- Passband: w/ modulation

Modulation: Change some attributes of a sinusoidal carrier waveform based on
- info bearing waveform → analog
- discrete info → digital

carrier: $\cos 2\pi f_c t$

1) ASK: Amplitude Shifting Keying

$s_i(t) = A_i \cos 2\pi f_c t$

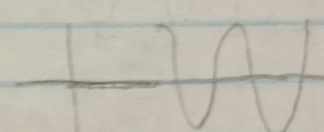
↳ $1 \ldots M$

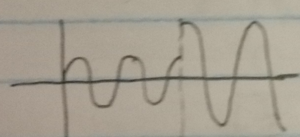
M-ary

Special Case: Binary: OOK (on off Keying)

$0 \rightarrow A_0 = 0$

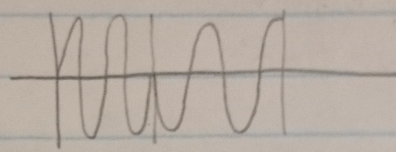
$1 \rightarrow A_1$





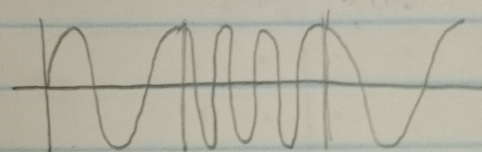
2) Phase Shifting Keying (PSK)

$$s_i(t) = A\cos(2\pi f_c t + \phi_i)$$

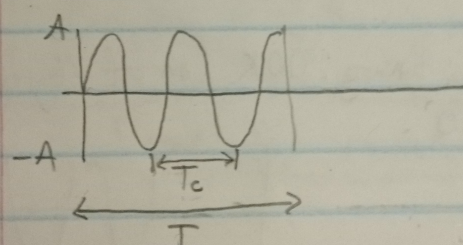


3) Frequency Shifting Keying (FSK)

$$s_i(t) = A\cos(2\pi f_i t)$$



Carrier: $A\cos 2\pi f_c t \qquad T_c = \frac{1}{f_c}$



T: signal duration

$*T >> T_c$

$f_c = 1\text{ GHz} \Rightarrow T_c = 1\text{ nsec}$

$R = 1$ M symbol/sec

$T = 1\ \mu\text{sec}$

$* \ \frac{T}{T_c} = \frac{T}{1/f_c} = Tf_c$: integer

$(\cos 2\pi f_c t, \sin 2\pi f_c t)$

$$\triangleq \int_0^T \underbrace{\cos 2\pi f_c t \sin 2\pi f_c t}_{2 \sin 2\cdot 2\pi f_c t} \, dt$$

$$= 2 \int_0^T \sin 2\cdot 2\pi f_c t$$

$$= 2 \times -\cos 2\cdot 2\pi f_c t \Big|_0^T$$

$$= -2\left(\cos 2\cdot 2\pi f_c T - \cos 0\right)$$

$$= 0$$

$* \cos 2\pi f_c t \perp \sin 2\pi f_c t$

$$\|\cos 2\pi f_c t\| = \sqrt{\int_0^T \cos^2 2\pi f_c t \, dt}$$

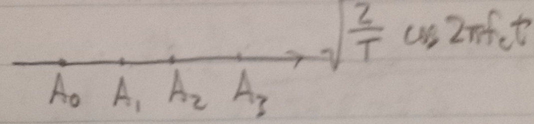
$$= \sqrt{\int_0^T \frac{1+\cos 2\cdot 2\pi f_c t}{2} \, dt}$$

$$= \sqrt{\underbrace{\tfrac{1}{2}\left[\int_0^T dt\right.}_{\sqrt{\frac{T}{2}}} + \left.\underbrace{\int_0^T \cos 2\cdot 2\pi f_c t \, dt}_{\frac{\sin 2\cdot 2\pi f_c t}{2\cdot 2\pi f_c}\Big|_0^T}\right]}$$

$$\left.\begin{array}{l} \phi_1(t) = \sqrt{\frac{T}{2}} \cos 2\pi f_c t \\ \phi_2(t) = \sqrt{\frac{T}{2}} \sin 2\pi f_c t \end{array}\right\} \text{orthoganal basis functions}$$

1) ASK $\quad S_i(t) = A_i \cos 2\pi f_c t \; : 1D$

Ex:
4-ary ASK



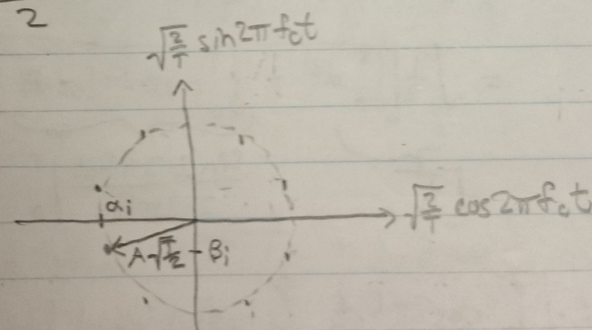
2) Phase Shift Keying (PSK) : 2D

$$s_i(t) = A\cos(2\pi f_c t + \theta_i)$$

$$s_i(t) = \underbrace{A\cos\theta_i}_{\alpha_i} \cos 2\pi f_c t - \underbrace{A\sin\theta_i}_{\beta_i} \sin 2\pi f_c t$$

$$\begin{aligned} \alpha_i^2 + \beta_i^2 &= A^2\cos^2\theta_i + B^2\sin^2\theta_i \\ &= A^2(\cos^2\theta + \sin^2\theta) \\ &= A^2 \end{aligned}$$

$$E_i = \frac{A^2 T}{2}$$



Special Case: FSK: Multi-D

Ex: f_i's can be chosen in such a way that

$\cos 2\pi f_i t \perp \cos 2\pi f_j t, \; i \neq j \; \forall_i$

M-ary $\rightarrow$ M Dimensional

Modulation Schemes Used in Practice

hybrid of ASK, PSK, FSK

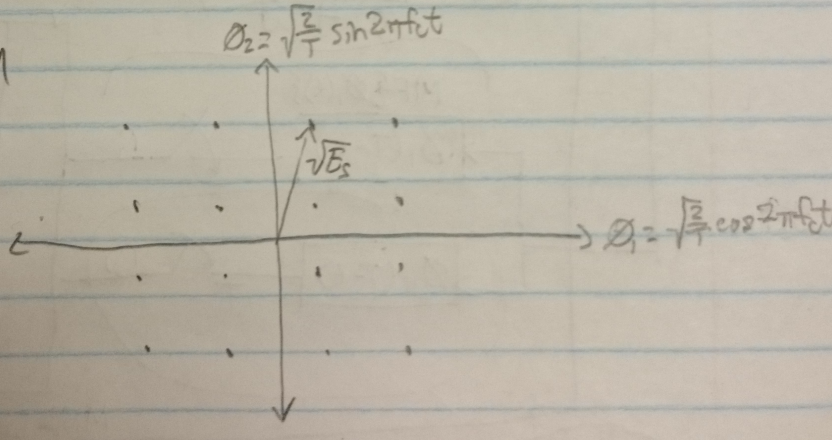
The most common ASK + PSK

$s_i(t) = A_i \cos(2\pi f_c t + \theta_i)$

QAM

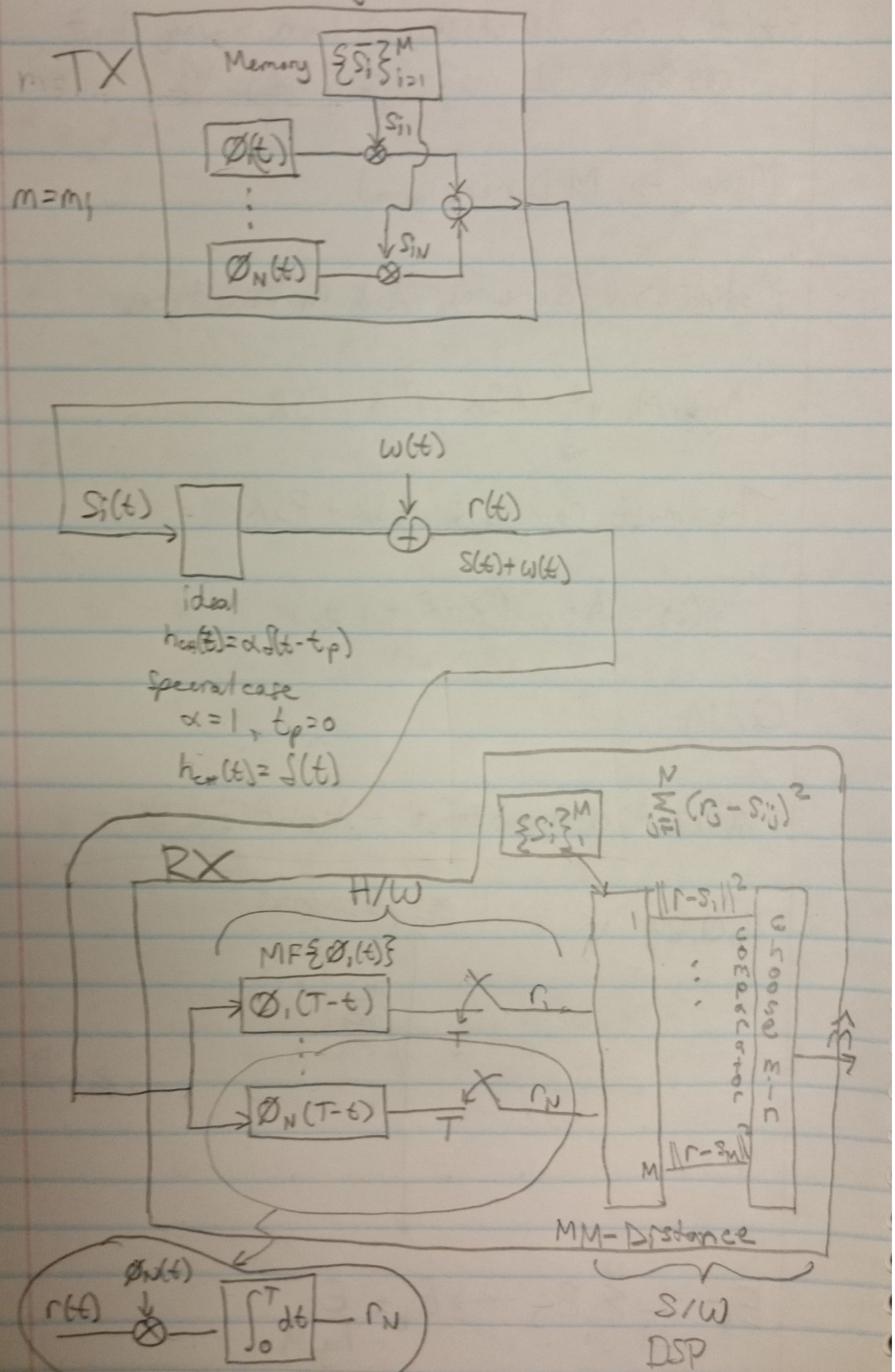
$s_i(t) = A_i \cos\theta_i \cos 2\pi f_c t - A_i \sin\theta_i \sin 2\pi f_c t$

16QAM



$E_s = \frac{1}{M} \sum_{j=1}^{M} E_j \longrightarrow E_b = \frac{E_s}{\log_2 M}$

Transceiver Design

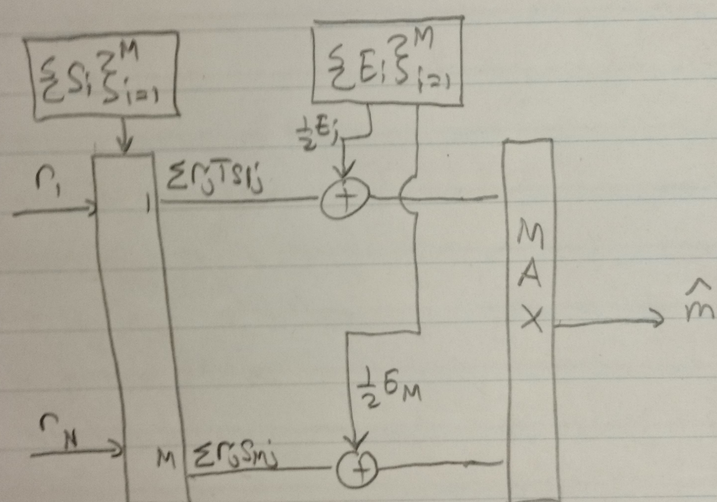


Minimum Distance = Max Correlation

$$\|\bar{r} - \bar{s}_i\|^2 \triangleq \sum_{j=1}^{N} (r_j - s_{ij})^2$$

$$= \underbrace{\sum_{j=1}^{N} r_j^2}_{\|r\|^2} - 2 \underbrace{\sum_{j=1}^{N} r_j s_j}_{\substack{(r(t), s_i(t)) \\ r_j T s_{ij}}} + \underbrace{\sum_{j=1}^{N} s_{ij}^2}_{\|s_i\|^2}$$

$$\max \; r_j T s_{ij} - \frac{1}{2} E_i$$



MAP

argMax $P(m_i \mid \bar{r})$