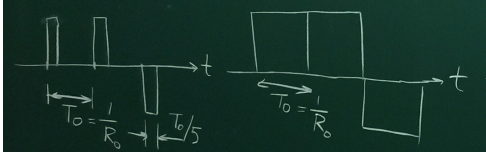
* Minimizing transmission BW

Nov 1/2016

Assume $R = R_0$ bits/sec

Consider $h_{Tx}(t)$, bit stream 1,1,-1

TX I



$BW \sim 5R_0$ Hz

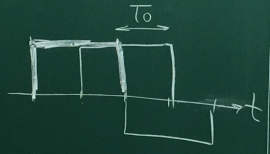
$E_{b,I} = \frac{1}{5} E_{b,II}$

Compromise: more Tx power → more expensive power amplifier

TX II

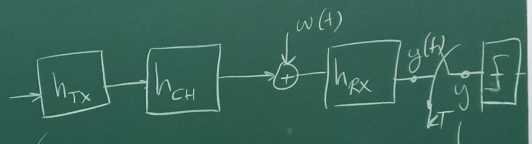
$BW \sim R_0$ Hz

TX III

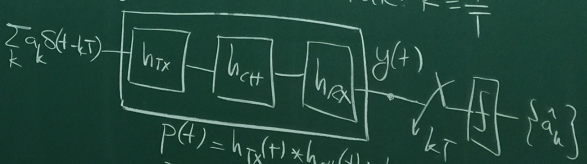


$BW \sim \frac{R_0}{2}$ Hz

Problem: ISI



- w(t) (→ MF: irrelevant)
- no restrictions on h_{Tx}, h_{CH}, h_{Rx}
- M-ary PAM
- Rate: $R = \frac{1}{T}$



$p(t) = h_{Tx}(t) * h_{CH}(t) * h_{Rx}(t)$

$P(f) = H_{Tx}(f) H_{CH}(f) H_{Rx}(f)$

Find structure of $p(t)$ such that

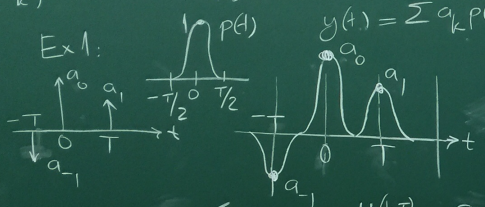
* no ISI
* min BW

$\{a_k\}$

$y(0) = \sum a_k p(-kT)$

$= \underbrace{\cdots + a_{-2} p(2T) + a_{-1} p(T)}_{\text{ISI}} + \underbrace{(a_0 p(0))}_{\text{desired}} + \underbrace{a_1 p(-T) + a_2 p(-2T) + \cdots}_{\text{ISI}}$

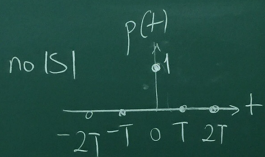
Ex 1:



$y(t) = \sum a_k p(t - kT)$

no ISI ✓

$y(kT) = \ldots, a_1, a_0, a_1, \ldots$



no ISI

$\sum_k \delta(t - kT) \times p(t) = \delta(t)$

$FT\left\{\sum \delta(t - kT)\right\} * P(f) = 1$

$\frac{1}{T} \sum \delta\left(f - \frac{k}{T}\right) * P(f) = 1$

$\boxed{\sum P\left(f - \frac{k}{T}\right) = T}$

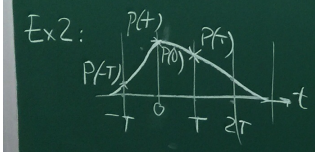
no ISI criterion in freq domain

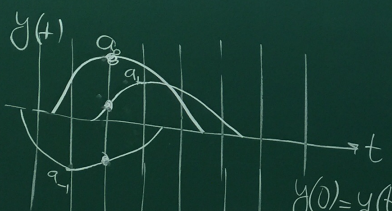
infinitely many options

$p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = kT \end{cases}$

no ISI condition in time domain

Ex 2:





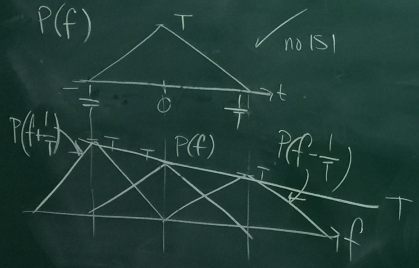
$y(0) = y(t=0) = \underbrace{a_{-1} P(T)}_{\text{ISI}} + \underbrace{(a_0 P(0))}_{\text{desired}} + \underbrace{a_1 P(-T)}_{\text{ISI}}$

1928, Nyquist criteria for no ISI

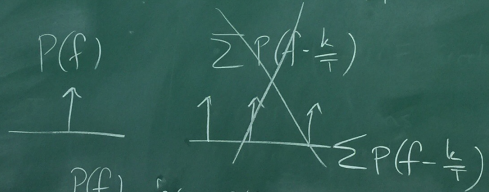
$$\sum_k P\left(f-\frac{k}{T}\right) = T$$

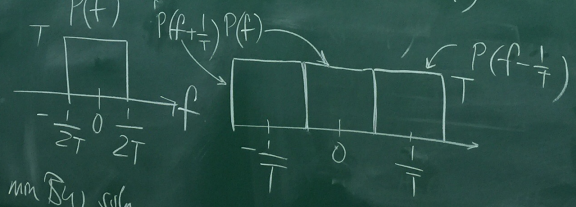
$$= \cdots + P\left(f+\frac{1}{T}\right) + P(f) + P\left(f-\frac{1}{T}\right) + \cdots = T$$

Ex 1:

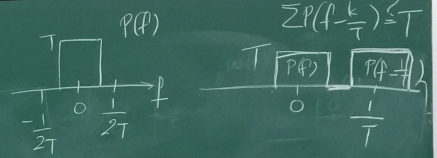


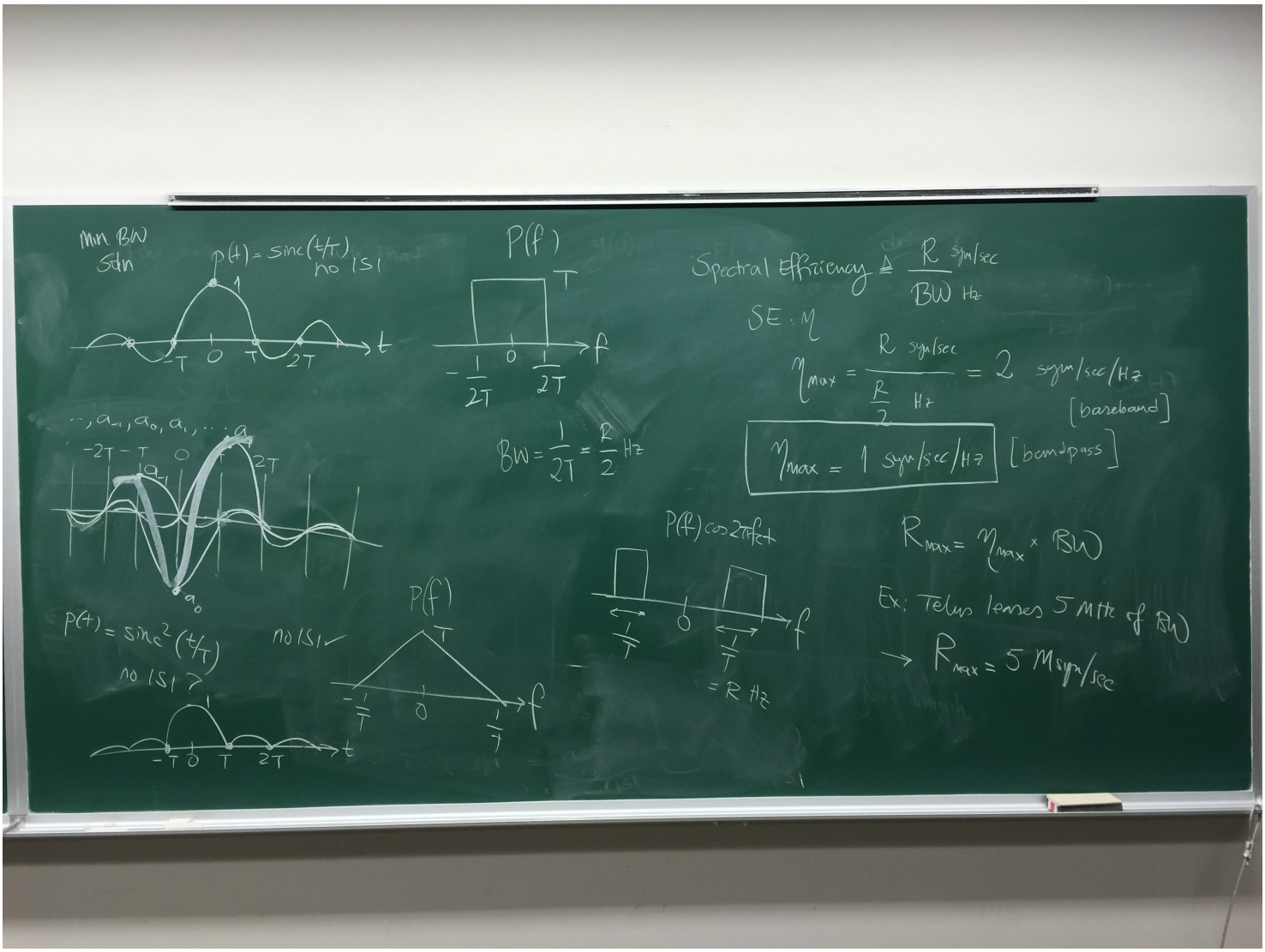
Which P(f) results in min BW?





min BW sinc




Min BW
Sdn
p(t) = sinc(t/T)
no ISI
t
-T
0
T
2T
P(f)
T
f
-1/2T
1/2T
Spectral Efficiency ≜ R sym/sec / BW Hz
SE: η
η_max = R sym/sec / R/2 Hz = 2 sym/sec/Hz
[baseband]
η_max = 1 sym/sec/Hz [bandpass]
BW = 1/2T = R/2 Hz
..., a_-1, a_0, a_1, ... a
-2T
-T
0
T
2T
a_1
a_0
P(f) cos 2πf_c t
f
1/T
R Hz
R_max = η_max × BW
Ex: Telus leases 5 MHz of BW
→ R_max = 5 Msym/sec
p(t) = sinc²(t/T)
no ISI ✓
no ISI ?
P(f)
T
f
-1/T
0
1/T