Sensitivity of pair-drive EIT in circular domains

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Abstract: EIT systems typically stimulate and measure the body across pairs of electrodes. The optimum electrode configuration to maximize sensitivity and resolution has been considered in several papers. We derive analytic expressions for the sensitivity and its spatial derivatives in order to help give insight into system design.

1 Introduction

Most EIT devices are pair-drive: pairs of electrodes are used to stimulate and measure from the body. There has been some discussion in the EIT literature about the best choice of angle between the driving electrodes (also called the "skip" pattern, referring to the number of electrodes "skipped" between the active pairs). This literature [1, 2, 3] has concluded that EIT sensitivity improves dramatically with pair-drive angle. However, this improved sensitivity appears to be at the expense of resolution, and larger skip patterns have less ability to resolve the two lungs. One proposed explanation of this effect is that the resolution depends both on the sensitivity and its spatial derivative [4].

In order to further explore this issue, this paper develops analytic expressions for the sensitivity of pair-drive EIT, from which an intuition of the compromises can be determined.

2 Sensitivity Calculations

The pair-drive configuration is illustrated below (Fig 1).

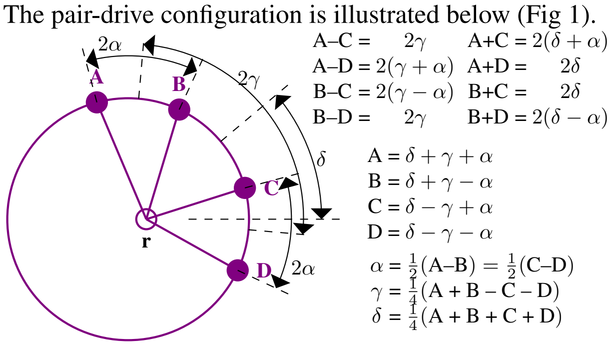

Figure 1: Unit radius circular or cylindrical domain with electrodes. Sensitivity at point, r (location (x, y)) is calculated, using drive pair (A→B) and measurement pair (C→D).

Initially, consider monopolar stimulation and measurement (stimulation with one electrode, S, and measurement with one electrode, M) and denote the sensitivity J, using an adjoint-field formulation

$$J_{\mathrm{r;S\to M}} \propto \nabla V_S(\mathrm{r}) \cdot \nabla V_M(\mathrm{r}) = \frac{\vec{\mathrm{r}}_\mathrm{S}}{\|\mathrm{r}_\mathrm{S}\|^d} \cdot \frac{\vec{\mathrm{r}}_\mathrm{M}}{\|\mathrm{r}_\mathrm{M}\|^d} \tag{1}$$

where $\vec{\mathrm{r}}_\mathrm{S} = \vec{\mathrm{r}} - \vec{\mathrm{S}}$, $\vec{\mathrm{r}}_\mathrm{M} = \vec{\mathrm{r}} - \vec{\mathrm{M}}$, and d is the dimension of the model (2 or 3 D). The equation simplifies at the centre $\mathrm{r} = 0$, $J_\mathrm{r} \propto \cos(\mathrm{S} - \mathrm{M})$; notation is (ab)used to consider points to be angles or vectors.

Pair-drive and measurement may be derived from the monopolar case, using $J_\mathrm{r} = J_{\mathrm{r;A\to C}} - J_{\mathrm{r;A\to D}} - J_{\mathrm{r;B\to C}} + J_{\mathrm{r;B\to D}}$. Sensitivity for r at the centre is:

$$\begin{aligned} S_{\mathrm{r}=0} &= \cos(\mathrm{A–C}) - \cos(\mathrm{A–D}) - \cos(\mathrm{B–C}) + \cos(\mathrm{B–D}) \\ &= \cos(2\gamma)\sin^2(\alpha) \end{aligned} \tag{2}$$

in terms of the angles defined in Fig 1.

3 Spatial derivatives of sensitivity

Equations for the spatial derivatives, ∇J, are more complicated. We show $\frac{\partial}{\partial x} J_\mathrm{r}$; $\frac{\partial}{\partial y} J_\mathrm{r}$ is obtained by rotation of δ.

$$\begin{aligned} \tfrac{\partial}{\partial x} J_{\mathrm{r;S\to M}} \propto\ & \frac{\mathrm{r}_{\mathrm{S},x} + \mathrm{r}_{\mathrm{M},x}}{(\|\mathrm{r}_\mathrm{S}\|\|\mathrm{r}_\mathrm{M}\|)^d} \\ & - d(\vec{\mathrm{r}}_\mathrm{S} \cdot \vec{\mathrm{r}}_\mathrm{M}) \frac{\mathrm{r}_{\mathrm{S},x}\|\mathrm{r}_\mathrm{M}\|^2 + \mathrm{r}_{\mathrm{M},x}\|\mathrm{r}_\mathrm{S}\|^2}{(\|\mathrm{r}_\mathrm{S}\|\|\mathrm{r}_\mathrm{M}\|)^{d+2}} \end{aligned} \tag{3}$$

At the centre, $\vec{\mathrm{r}} = 0$:

$$\nabla J_{\mathrm{r}=0} \propto \left(\cos(\mathrm{S–M}) - \tfrac{1}{d}\right) \begin{bmatrix} \cos(\mathrm{S}) + \cos(\mathrm{M}) \\ \sin(\mathrm{S}) + \sin(\mathrm{M}) \end{bmatrix} \tag{4}$$

For pair drive, we can calculate

$$\nabla J_{\mathrm{r}=0} \propto \begin{bmatrix} \cos(\delta) \\ \sin(\delta) \end{bmatrix} \cos(3\gamma)\sin^2(\alpha)\cos(\alpha) \tag{5}$$

Finally, the ratio of spatial derivative to sensitivity is

$$\frac{\|\nabla J_\mathrm{r}\|}{J_\mathrm{r}} \propto \frac{\cos(3\gamma)}{\cos(2\gamma)} \cos(\alpha) \tag{6}$$

4 Discussion

It is hoped that analytic expressions for pair-drive sensitivity provide insight into the compromises involved in EIT systems design.

The formulae for pair-drive sensitivity take a surprisingly simple form and are a function of simple trigonometric expressions for the central point in a cylindrical domain. In a full pair-drive system, γ and δ are rotated around and so will integrate to unity. The remaining factor is the sensitivity to the "skip" distance, α. Because of the additional $\cos(\alpha)$ factor, the resolving component of ∇J decreases as skip increases. Antother interesting effect is the $\cos(3\gamma)$ vs $\cos(2\gamma)$, although it's unclear how this affects a full EIT measurement.

References

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