# Errata

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**Title** : A Formulation of the Potential for Communication Condition using C$^2$KA
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**Location**: Section 2.2, Definition 4(ii), Page 164

**Description**: There is an error in Definition 4(ii) that causes unintended consequences of the axiomatisation of C$^2$KA.

**Correction**:

**Definition 4** (Communicating Concurrent Kleene Algebra)**.** *A Communicating Concurrent Kleene Algebra* (C$^2$KA) *is a system* $\big(\mathscr{S}, \mathscr{K}\big)$*, where* $\mathscr{S} = \big(S, \oplus, \odot, \mathfrak{d}, \mathfrak{n}\big)$ *is a stimulus structure and* $\mathscr{K} = \big(K, +, *, ;, ^{\circledast}, ^{\odot}, 0, 1\big)$ *is a* CKA *such that* $\big({}_{\mathscr{S}}K, +\big)$ *is a unitary and zero-preserving* left $\mathscr{S}$-semimodule *with mapping* $\circ : S \times K \to K$ *and* $\big(S_{\mathscr{K}}, \oplus\big)$ *is a unitary and zero-preserving* right $\mathscr{K}$-semimodule *with mapping* $\lambda : S \times K \to S$*, and where the following axioms are satisfied for all* $a, b, c \in K$ *and* $s, t \in S$*:*

*(i)* $s \circ (a \,;b) = (s \circ a) \,; \big(\lambda(s, a) \circ b\big)$

*(ii)* $a \leq_{\mathscr{K}} c \;\vee\; b = 1 \;\vee\; (s \circ a) \,; \big(\lambda(s, c) \circ b\big) = 0$

*(iii)* $\lambda(s \odot t, a) = \lambda\big(s, (t \circ a)\big) \odot \lambda(t, a)$

In Definition 4, Axiom (ii), which is referred to as the *cascading output law*, states that when an external stimulus is introduced to the sequential composition $(a \,; b)$, then either the cascaded stimulus must be generated by the behaviour $a$, or the behaviour $b$ must be the idle agent behaviour 1. It allows distributivity of $\circ$ over $;$ to be applied indiscriminately and ensures consistency between the next behaviour and next stimulus mappings with respect to the sequential composition of agent behaviours.

**Location**: Section 3.1, Proposition 3(ii), Page 166

**Description**: There is an error in the proof of Proposition 3(ii) (see below) causing the condition for Proposition 3(ii) to be incorrect.

**Correction**:

**Proposition 3.** *Let* $\mathsf{A} = \langle a \rangle$, $\mathsf{B} = \langle b \rangle$, *and* $\mathsf{C} = \langle c \rangle$ *be agents in* $\mathscr{C}$.

*(i)* *If* $\mathsf{B} \rightarrow_{\mathscr{S}} \mathsf{C}$ *then* $(\mathsf{A} + \mathsf{B}) \rightarrow_{\mathscr{S}} \mathsf{C}$.

*(ii)* *If* $\mathsf{A} \rightarrow_{\mathscr{S}} \mathsf{B}$ *then* $\mathsf{A} \rightarrow_{\mathscr{S}} (\mathsf{B} + \mathsf{C})$ *if* $\forall\big(s,t \ \mid\ s,t \in S_b \ \wedge\ t \leq_{\mathscr{S}} \lambda(s,a) \ :\ \neg(t \circ b \leq_{\mathscr{K}} b + c \ \wedge\ t \circ c \leq_{\mathscr{K}} b + c)\big)$.

*Proof.* The proof of (i) uses the definition of $\rightarrow_{\mathscr{S}}$, the distributivity of $\lambda$ over $+$, the definition of $\leq_{\mathscr{S}}$, and the fact that $\oplus$ is left-isotone with respect to $\leq_{\mathscr{S}}$. The proof of (ii) involves the definition of $\rightarrow_{\mathscr{S}}$, involves monotonic $\exists$-body, anti-monotonic $\neg$, distributivity of $\circ$ over $+$, and substitution of $=$ by $=$. $\square$

**Location**: Section 3.3, Proposition 5(i) and (ii), Page 168

**Description**: As a result of the error in Proposition 3(ii), there is an error in the proof and formulation of Proposition 5(i) and (ii). A slight change in the formulation of Proposition 5 is also made in order to simplify the proofs.

**Correction**:

**Proposition 5.** *Let* $\mathsf{A} \leadsto^* \mathsf{B}$ *such that* $\exists\big(\mathsf{C} \ \mid\ \mathsf{C} \in \mathscr{C} \ :\ \mathsf{A} \leadsto \mathsf{C} \ \wedge\ \mathsf{C} \leadsto \mathsf{B}\big)$ *where* $\mathsf{A} = \langle a \rangle$, $\mathsf{B} = \langle b \rangle$, *and* $\mathsf{C} = \langle c \rangle$. *Let* $\mathrm{R}$ *be the given dependence relation. Suppose* $\mathsf{C}$ *is replaced by another agent* $\mathsf{C}' = \langle c' \rangle$. *Then,*

*(i)* *If* $c' = (c \,;d)$, *then* $\mathsf{A} \leadsto^* \mathsf{B}$ *if* $\forall\big(s,t \mid s,t \in S_b \wedge t \leq_{\mathscr{S}} \lambda(s,c) : \lambda(t,d) = t\big) \ \vee\ (c\,;d)\,\mathrm{R}\,b$.

*(ii)* *If* $c' = (c + d)$, *then* $\mathsf{A} \leadsto^* \mathsf{B}$ *if* $\forall\big(s,t \ \mid\ s,t \in S_b \ \wedge\ t \leq_{\mathscr{S}} \lambda(s,a) \ :\ \neg(t \circ c \leq_{\mathscr{K}} c + d \ \wedge\ t \circ d \leq_{\mathscr{K}} c + d)\big)$.

*(iii)* *If* $c' = c^{\textcircled{\scriptsize !}}$, *then* $\mathsf{A} \leadsto^* \mathsf{B}$.

*(iv)* *If* $c' = 0$ *or* $c' = 1$ *and the* $\mathrm{C^2KA}$ *is without reactivation, then* $\neg(\mathsf{A} \leadsto^* \mathsf{B})$.

*(v)* *If* $c' \in \mathrm{Orb_S}(c)$, *then* $\mathsf{A} \leadsto^* \mathsf{B}$.

*(vi)* *If* $c'$ *is a fixed point behaviour, then* $\mathsf{A} \leadsto^* \mathsf{B}$ *only if* $a\,\mathrm{R}\,c' \ \wedge\ c'\,\mathrm{R}\,b$.

*Proof.* Each of the proofs involve the applications of definitions of $\leadsto$, $\rightarrow_{\mathscr{S}}$, and $\rightarrow_{\mathscr{E}}$ as well as the basic axioms of $\mathrm{C^2KA}$. $\square$

**Location**: Appendix A, Detailed Proof of Proposition 3(ii), Page 172

**Description**: There is an error in the detailed proof of Proposition 3(ii).

**Correction**: Let $\mathsf{A} = \langle a \rangle$, $\mathsf{B} = \langle b \rangle$, and $\mathsf{C} = \langle c \rangle$ be agents in $\mathscr{C}$.

(ii) If $\mathsf{A} \rightarrow_{\mathscr{S}} \mathsf{B}$ then $\mathsf{A} \rightarrow_{\mathscr{S}} (\mathsf{B} + \mathsf{C})$ if $\forall\big(s,t \;\mid\; s,t \in S_b \;\wedge\; t \leq_{\mathscr{S}} \lambda(s,a) \;:\; \neg(t \circ b \leq_{\mathscr{K}} b + c \;\wedge\; t \circ c \leq_{\mathscr{K}} b + c)\big)$.

$$
\begin{aligned}
& \mathsf{A} \rightarrow_{\mathscr{S}} \mathsf{B} \implies \mathsf{A} \rightarrow_{\mathscr{S}} (\mathsf{B} + \mathsf{C}) \\
\Longleftrightarrow \quad & \langle \text{ Definition of } \rightarrow_{\mathscr{S}} \rangle \\
& \exists\big(s,t \mid s,t \in S_b \wedge t \leq_{\mathscr{S}} \lambda(s,a) : t \circ b \neq b\big) \implies \\
& \exists\big(s,t \mid s,t \in S_b \wedge t \leq_{\mathscr{S}} \lambda(s,a) : t \circ (b+c) \neq (b+c)\big) \\
\Longleftarrow \quad & \langle \text{ Monotonic } \exists\text{-Body } \rangle \\
& \forall\big(s,t \mid s,t \in S_b \wedge t \leq_{\mathscr{S}} \lambda(s,a) : t \circ b \neq b \implies t \circ (b+c) \neq (b+c)\big) \\
\Longleftarrow \quad & \langle \text{ Anti-monotonic } \neg \rangle \\
& \forall\big(s,t \mid s,t \in S_b \wedge t \leq_{\mathscr{S}} \lambda(s,a) : t \circ (b+c) = (b+c) \implies t \circ b = b\big) \\
\Longleftrightarrow \quad & \langle \text{ Distributivity of } \circ \text{ over } + \rangle \\
& \forall\big(s,t \mid s,t \in S_b \wedge t \leq_{\mathscr{S}} \lambda(s,a) : (t \circ b + t \circ c) = (b+c) \implies t \circ b = b\big) \\
\Longleftrightarrow \quad & \langle \text{ Idempotence of } + \rangle \\
& \forall\big(s,t \mid s,t \in S_b \wedge t \leq_{\mathscr{S}} \lambda(s,a) : (t \circ b + t \circ c) = (b+c+b+c) \implies \\
& t \circ b = b\big) \\
\Longleftarrow \quad & \langle \text{ Substitution of } = \text{ by } = \rangle \\
& \forall\big(s,t \mid s,t \in S_b \wedge t \leq_{\mathscr{S}} \lambda(s,a) : \big(t \circ b = (b+c) \wedge t \circ c = (b+c) \wedge \\
& (t \circ b + t \circ c) = (t \circ b + t \circ c)\big) \implies t \circ b = b\big) \\
\Longleftrightarrow \quad & \langle \text{ Reflexivity of } = \;\&\; \text{Identity of } \wedge \;\&\; \text{Definition of } \implies \rangle \\
& \forall\big(s,t \mid s,t \in S_b \wedge t \leq_{\mathscr{S}} \lambda(s,a) : \neg\big(t \circ b = (b+c) \wedge t \circ c = (b+c)\big) \vee \\
& t \circ b = b\big) \\
\Longleftrightarrow \quad & \langle \text{ De Morgan } \rangle \\
& \forall\big(s,t \mid s,t \in S_b \wedge t \leq_{\mathscr{S}} \lambda(s,a) : t \circ b \neq (b+c) \vee t \circ c \neq (b+c) \vee \\
& t \circ b = b\big) \\
\Longleftarrow \quad & \langle \text{ Hypothesis: } \forall\big(s,t \mid s,t \in S_b \wedge t \leq_{\mathscr{S}} \lambda(s,a) : \neg(t \circ b \leq_{\mathscr{K}} \\
& \quad b + c \wedge t \circ c \leq_{\mathscr{K}} b+c)\big) \;\;\&\;\; \text{Weakening: } P \implies P \vee Q \rangle \\
& \mathsf{true}
\end{aligned}
$$

**Location**: Appendix A, Detailed Proof of Proposition 5(i), Page 173

**Description**: There is an error in the detailed proof of Proposition 5(i).

**Correction**: Let $\mathsf{A} \rightsquigarrow^* \mathsf{B}$ such that $\exists\big(\mathsf{C} \mid \mathsf{C} \in \mathscr{C} \;:\; \mathsf{A} \rightsquigarrow \mathsf{C} \;\wedge\; \mathsf{C} \rightsquigarrow \mathsf{B}\big)$.

(i) $\mathsf{C}' = \big\langle c\,;d\big\rangle$

$$
\begin{array}{ll}
& \mathsf{A} \rightsquigarrow \mathsf{C}' \;\wedge\; \mathsf{C}' \rightsquigarrow \mathsf{B} \\
\Longleftrightarrow & \quad \langle \text{ Substitution: } \mathsf{C}' = (\mathsf{C}\,;\mathsf{D}) \text{ where } \mathsf{C} = \big\langle c\big\rangle \text{ and } \mathsf{D} = \big\langle d\big\rangle \;\rangle \\
& \mathsf{A} \rightsquigarrow (\mathsf{C}\,;\mathsf{D}) \;\wedge\; (\mathsf{C}\,;\mathsf{D}) \rightsquigarrow \mathsf{B} \\
\Longleftarrow & \quad \langle \text{ Hypothesis: } \mathsf{A} \rightsquigarrow \mathsf{C} \implies \mathsf{A} \rightsquigarrow (\mathsf{C}\,;\mathsf{D}) \quad \& \quad \text{Identity of } \wedge \;\rangle \\
& (\mathsf{C}\,;\mathsf{D}) \rightsquigarrow \mathsf{B} \\
\Longleftrightarrow & \quad \langle \text{ Definition of } \rightsquigarrow \;\rangle \\
& (\mathsf{C}\,;\mathsf{D}) \rightarrow_{\mathscr{S}} \mathsf{B} \;\vee\; (\mathsf{C}\,;\mathsf{D}) \rightarrow_{\mathscr{E}} \mathsf{B} \\
\Longleftrightarrow & \quad \langle \text{ Definition of } \rightarrow_{\mathscr{S}} \quad \& \quad \text{Definition of } \rightarrow_{\mathscr{E}} \;\rangle \\
& \exists\big(s,t \;\mid\; s,t \in S_b \;\wedge\; t \leq_{\mathscr{S}} \lambda(s,(a\,;c)) \;:\; t \circ b \neq b\big) \;\vee\; (c\,;d)\,\mathrm{R}\,b \\
\Longleftrightarrow & \quad \langle \text{ Distributivity of } \lambda \text{ over } ; \;\rangle \\
& \exists\big(s,t \;\mid\; s,t \in S_b \;\wedge\; t \leq_{\mathscr{S}} \lambda\big(\lambda(s,a),c\big) \;:\; t \circ b \neq b\big) \;\vee\; (c\,;d)\,\mathrm{R}\,b \\
\Longleftarrow & \quad \langle \text{ Hypothesis: } \big[\mathsf{C} \rightsquigarrow \mathsf{B} \;\wedge\; \big(\,\forall\big(s,t \mid s,t \in S_b \wedge t \leq_{\mathscr{S}} \lambda(s,c) : \\
& \quad \lambda(t,d) = t\big) \;\vee\; (c\,;d)\,\mathrm{R}\,b\big)\big] \;\rangle \\
& \mathsf{true}
\end{array}
$$