Some Good Block PSK Real Number Codes for Rayleigh Fading Channels

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Abstract — PSK real number codes for the Rayleigh fading channel were proposed in [1]. This paper tabulates the detailed mapping of some simple full-diversity low-delay codes in the original and the manually refined forms. The performance is examined by simulation at various interleaving depths. For the codes considered, the manually refined ones are found to be easy to implement and offer very little performance loss compared with the original.

I. Introduction

Optimum coded modulation design for mobile wireless channels which typically experience severe Rayleigh/Rician fading is a topic of continued research. The lengthy burst errors caused due to the occurrence of deep fades, particularly at slow mobile speeds, results in the performance of coded modulation schemes being worse than that of uncoded modulation, even when significant interleaving depths are employed.

For Rayleigh fading channels, it has been shown that, using coherent maximum likelihood sequence estimation with known channel information, the pairwise error probability of the coded modulation schemes depends greatly on the degree of time-diversity [2-4] instead of squared Euclidean distance. Although many BCM codes for Rayleigh fading channels [e.g. 5-9] have already been designed based on criteria such as increased diversity, their diversity length is still less than the codeword length. To achieve a given diversity length, longer channel codewords are used, which leads to increased interleaving delay.

Block real number coding was introduced in [1] as a generalization of Block Coded Modulation (BCM). It uses a new design method and more accurate design criteria to search for the optimal code in the expanded code space. The codes so obtained are found to be able to provide full diversity for the Rayleigh fading channel.

In this paper, we choose a few good full diversity low-delay low-complexity codes found in [1] for further investigation. The detailed mapping tables of the codes are presented; the manual refinements of the numerical solution are conducted; the performance of the codes are simulated for different interleaving depths in the Rayleigh faded channel.

II. Real Number Code and Its Design

This section briefly reviews the concept of the real number code and its design method, proposed in [1].

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An (n,k) block real number code transmits k information bits by n 2-dimensional channel symbols with coding rate $r \triangleq k/n$ bits/$2D$. Its encoder is shown in Fig. 1. The encoding operation is simply mapping each k-long binary information block $\boldsymbol{b} \triangleq (b_1, b_2, ..., b_j, ..., b_k) \in \boldsymbol{B}$ by a memoryless one-to-one correspondence to a $2n$ dimensional real vector $\boldsymbol{s} \triangleq (x_1, y_1, x_2, y_2, ..., x_l, y_l, ..., x_n, y_n) \in \boldsymbol{S}$, where the set $\boldsymbol{B} \triangleq \{(b_1, b_2, ..., b_j, ..., b_k) : b_j=\{0,1\}, j=1,2,...,k\}$, and the set $\boldsymbol{S} \triangleq \{s^{(i)} \triangleq (x_1^{(i)}, y_1^{(i)}, ..., x_l^{(i)}, y_l^{(i)}, ..., x_n^{(i)}, y_n^{(i)}) : s^{(i)} \in \boldsymbol{R}^{2n},\ i = 1, 2, ..., M\}$, $M \triangleq 2^k$. The real vector $\boldsymbol{s}$ is the final channel codeword sent through a $2n$-dimensional equivalent channel, which can be constructed, e.g., by a 2-dimensional physical channel and a pair of parallel-to-serial and serial-to-parallel converters on either side. As there are in total $M=2^k$ combinations of input bits, M points in the $2n$-dimensional real space $\boldsymbol{R}^{2n}$ will be chosen as legal codewords; they are denoted as $s^{(i)}$, $i=1,2,...,M$.

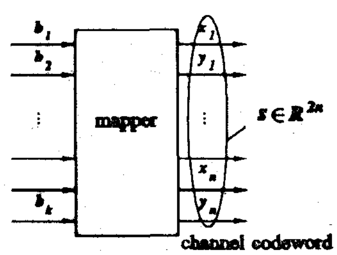

Fig. 1

$(x_l^{(i)}, y_l^{(i)})$ are then the coordinates of the $2D$ signal point of the ith codeword at time slot l. In the receiver, the noisy and distorted counterpart of $\boldsymbol{s}$ can be detected block by block by a maximum likelihood detector. The *codeword length* and the *diversity length* of the code are defined as the value of n and the minimum Hamming distance between the codewords respectively .

It is clear that the encoder mapping must be designed optimally so as to achieve the best performance in some sense, and this task is in fact to choose the real number values of $2nM$ elements in $s^{(i)}$, $i=1,2,...,M$. In the design, we choose the criterion to be the minimization of the union-Chernoff upper bound of the word error rate [2,3]. The code design problem can be transferred to a multivariable constrained nonlinear optimization problem [1] and solved numerically. The optimization model is formulated as

$$\min_{\substack{s^{(m)} \in R^{2n} \\ m=1,2,...,M}} \left\{ \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \prod_{l=1}^{n} \left[1 + \frac{E_s}{4N_0} \left[(x_l^{(i)} - x_l^{(j)})^2 + (y_l^{(i)} - y_l^{(j)})^2 \right] \right]^{-1} \right\}$$

subject to

$$[x_l^{(i)}]^2 + [y_l^{(i)}]^2 = 1 \text{ for } i=1,2,...,M;\ l=1,2,...,n$$

Following the new design approach, the code is constructed

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by a one-step mapping, rather than through some intermediate binary codes; however, the conventional BCM PSK codes are in fact still observed to be within its search space as special cases.

III. The Codes and Their Performance

This section discusses five codes and their simplified versions. All the codes are full diversity ones, i.e., the diversity length equals the codeword length.

In the simulation of the code performance, the system assumed is as follows: the coded channel block is converted to serial form and transmitted as a sequence of 2*D* symbols; the sequence is interleaved by a block-type symbol interleaver, then transmitted through the slow flat Rayleigh fading channel; in the receiver, the received sequence in deinterleaved accordingly and detected by a maximum likelihood block decoder with known channel state information and perfect timing [2-4]. The block-type interleaver (/deinterleaver) is realized by a n by m matrix which is written column by column (/row by row) and read row by row (/column by column), where n is the codeword length. The *interleaving depth* is defined as $n\times m$, which determines the total interleaving delay; the *interleaving degree* is defined as the dimension m of the matrix. In all the simulations, the normalized Doppler frequency $f_d\,T_s$ is assumed to be 0.001.

1. Length 2 Codes

(2,2) code (r= 1 bit/2*D*): The original code mapping table obtained from the numerical solution of the optimization search is given in Table I (a). In the table, the first column indicates the codewords, each corresponds to one of the four combinations of 2-bit information block by certain one-to-one mapping rule; the second column lists the corresponding signal point coordinates of the two 2*D* channel symbols mapped to. The two 2*D* constellations are unequally spaced 4-ary PSK, they are also depicted in Fig. 2.

Table I (a): The mapping of the original (2,2) code

Code-words	Codeword coordinates	
	1st 2D	2nd 2D
$s^{(0)}$	(9.5051E-1,-3.1176E-1)	(4.3326E-1, 9.0170E-1)
$s^{(1)}$	(1.7443E-1, 9.8501E-1)	(-9.5384E-1, 3.0161E-1)
$s^{(2)}$	(-9.5051E-1, 3.1176E-1)	(9.5384E-1,-3.0161E-1)
$s^{(3)}$	(-1.7443E-1,-9.8501E-1)	(-4.3326E-1,-9.0170E-1)

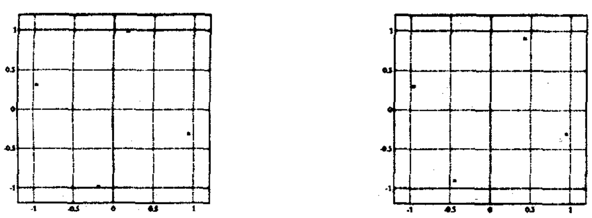

Fig. 2

Table I (b): Refined (2,2) code

Code-words	Phase number of codewords	
$s^{(0)}$	0	0
$s^{(1)}$	1	1
$s^{(2)}$	2	3
$s^{(3)}$	3	2

The constellations are observed to be almost equally spaced. It can be expected that the performance would not change much if the signal point locations are replaced by the equally spaced QPSK accordingly. By doing so, the mapping table of the manually refined code is obtained as in Table I (b), where a phase number i (i=0, 1, ..., 2^k-1) means the signal point has a phase of $2\pi i/2^k$, here k=2 for this code.

Fig. 3 shows the simulated word error rate of the original code (denoted by the abbreviation "O.") and the refined code (denoted by "R.") at various interleaving degrees. The union-Chernoff upper bound [2,3] of the original and the refined codes as well as the exact uncoded BPSK performance are also plotted for comparison. It can be observed that: 1) the refined code performs almost as well as the original in this case; 2) for good performance, the interleaving degree should be 90 or more at the simulated fading speed.

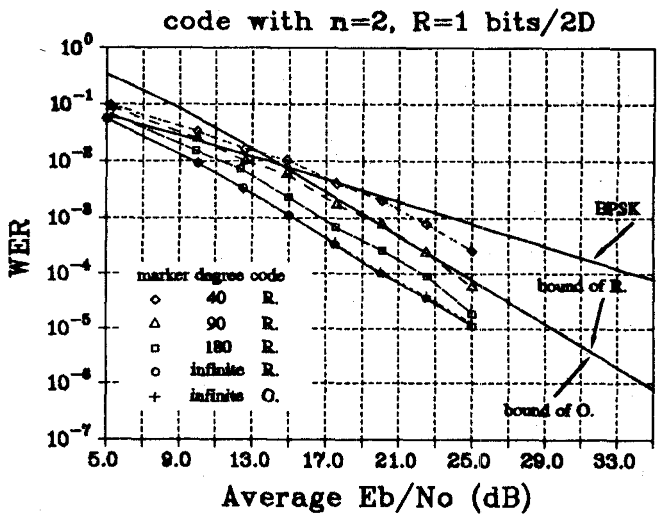

Fig. 3

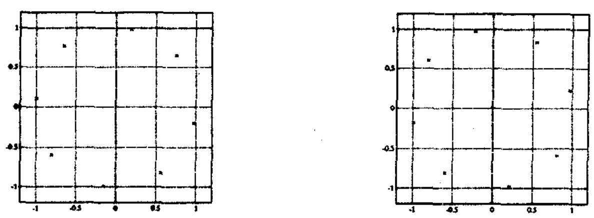

Fig. 4

(2,3) code (r = 1.5 bits/2*D*): The mapping table of the original and refined codes are given in Tables II (a) and (b) respectively. The original code is an 8-ary unequally spaced

Table II (a): The mapping of the original (2,3) code

Code-words	Codeword coordinates	
	1st 2D	2nd 2D
$s^{(0)}$	(5.6985E-1,-8.2221E-1)	(8.0991E-1,-5.8713E-1)
$s^{(1)}$	(9.8132E-1,-1.9435E-1)	(-2.2708E-1, 9.7422E-1)
$s^{(2)}$	(7.6529E-1, 6.4427E-1)	(9.7552E-1, 2.2143E-1)
$s^{(3)}$	(1.9742E-1, 9.8070E-1)	(-5.9292E-1,-8.0568E-1)
$s^{(4)}$	(-6.4188E-1, 7.6730E-1)	(-8.0223E-1, 5.9758E-1)
$s^{(5)}$	(-9.9487E-1, 1.0481E-1)	(2.1442E-1,-9.7709E-1)
$s^{(6)}$	(-7.9899E-1,-6.0198E-1)	(5.5565E-1, 8.3182E-1)
$s^{(7)}$	(-1.4414E-1,-9.8994E-1)	(-9.8459E-1,-1.7681E-1)

Table II (b): Refined (2,3) code

Code-words	Phase	number of codewords
$s^{(0)}$	0	0
$s^{(1)}$	1	3
$s^{(2)}$	2	1
$s^{(3)}$	3	6
$s^{(4)}$	4	4
$s^{(5)}$	5	7
$s^{(6)}$	6	2
$s^{(7)}$	7	5

PSK one, the constellations are given in Fig. 4.

Fig. 5 gives the simulated word error performance together with the bounds of the original and the refined codes. The performance loss due to the refinement is nearly invisible and the complexity refined codes is greatly reduced by using standard 8PSK constellations.

<u>(2,4) code (r = 2 bits/2D):</u> The mapping table of the original and refined codes are given in Tables III (a) and (b) respectively. The original code has unequally spaced PSK constellations of size 16, the constellations are given in Fig. 6. Simulation results of the original and the refined codes are presented in Fig. 7 with the simulated performance of uncoded QPSK for comparison. The union-Chernoff bounds are also plotted for both codes. Again, the refinement causes very little loss.

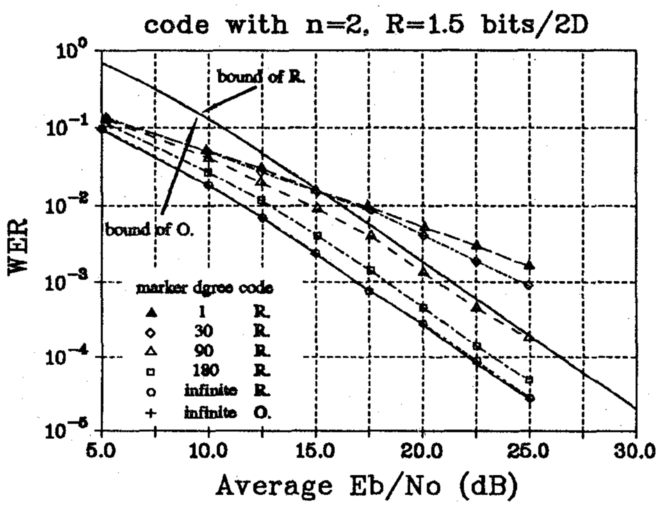

Fig. 5

Table III (a): The mapping of the original (2,4) code

Code-words	Codeword coordinates	
	1st 2D	2nd 2D
$s^{(0)}$	(1.0038E+0,-4.0075E-2)	(9.7534E-1,-2.2921E-1)
$s^{(1)}$	(9.3495E-1, 3.6754E-1)	(-1.9089E-1,-9.8356E-1)
$s^{(2)}$	(7.0547E-1, 7.1520E-1)	(-8.4535E-1, 5.3778E-1)
$s^{(3)}$	(3.7269E-1, 9.3290E-1)	(9.7407E-1, 2.3456E-1)
$s^{(4)}$	(-4.8474E-3, 1.0046E+0)	(-6.1647E-1,-7.8981E-1)
$s^{(5)}$	(-3.4510E-1, 9.4346E-1)	(1.4177E-1, 9.9183E-1)
$s^{(6)}$	(-6.8469E-1, 7.3512E-1)	(8.4407E-1,-5.3979E-1)
$s^{(7)}$	(-9.0281E-1, 4.4062E-1)	(-9.8564E-1, 1.7984E-1)
$s^{(8)}$	(-1.0030E+0, 5.6614E-2)	(5.8402E-1, 8.1410E-1)
$s^{(9)}$	(-9.6960E-1,-2.6286E-1)	(1.3539E-1,-9.9272E-1)
$s^{(10)}$	(-7.1438E-1,-7.0632E-1)	(-8.4750E-1,-5.3439E-1)
$s^{(11)}$	(-3.3948E-1,-9.4550E-1)	(6.1197E-1,-7.9330E-1)
$s^{(12)}$	(-4.3053E-2,-1.0037E+0)	(-6.1018E-1, 7.9468E-1)
$s^{(13)}$	(3.8492E-1,-9.2793E-1)	(8.3920E-1, 5.4733E-1)
$s^{(14)}$	(7.0193E-1,-7.1868E-1)	(-9.8682E-1,-1.7323E-1)
$s^{(15)}$	(9.3333E-1,-3.7163E-1)	(-1.8703E-1, 9.8430E-1)

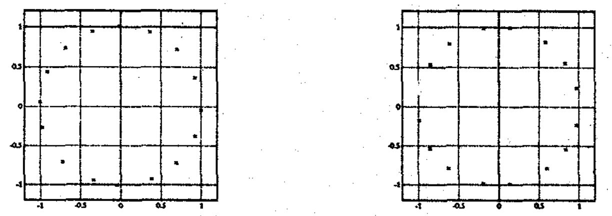

Fig. 6

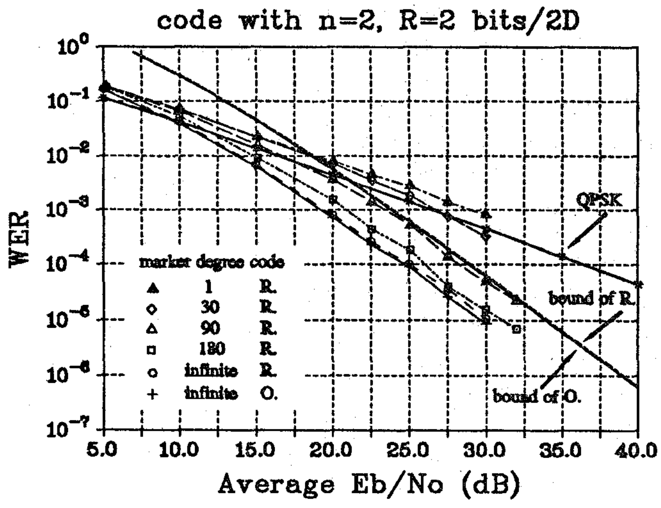

Fig. 7

2. A Length 3 Code

The length 3 code has coding rate of 1 bit/2D, so it is a (3,3) code. The mapping for the original code and the refined version are tabulated in Table IV (a) and (b) respectively. The three 2D PSK constellations of the original code are plotted in Fig. 8. Simulated performance and the bounds for the both code versions are illustrated in Fig. 9 with BPSK performance for comparison.

Table III (b):

Refined (2,4) code

Code-words	Phase number of codewords
$s^{(0)}$	0 0
$s^{(1)}$	1 12
$s^{(2)}$	2 7
$s^{(3)}$	3 1
$s^{(4)}$	4 11
$s^{(5)}$	5 4
$s^{(6)}$	6 15
$s^{(7)}$	7 8
$s^{(8)}$	8 3
$s^{(9)}$	9 13
$s^{(10)}$	10 10
$s^{(11)}$	11 14
$s^{(12)}$	12 6
$s^{(13)}$	13 2
$s^{(14)}$	14 9
$s^{(15)}$	15 5

Table IV (b):

Refined (3,3) code

Code-words	Phase number of codewords
$s^{(0)}$	0 0 0
$s^{(1)}$	1 6 3
$s^{(2)}$	2 4 6
$s^{(3)}$	3 1 4
$s^{(4)}$	4 5 1
$s^{(5)}$	5 2 7
$s^{(6)}$	6 7 5
$s^{(7)}$	7 3 2

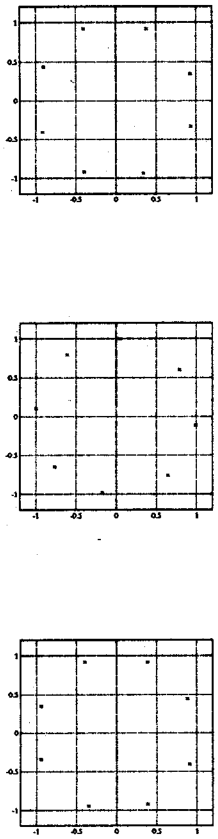

Fig. 8

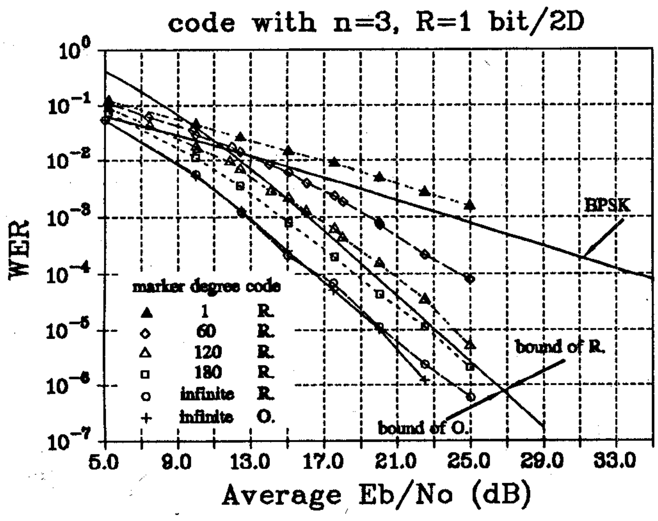

Fig. 9

3. *A Length 4 Code*

This is a (4,4) rate 1 bit/2*D* code. The original code obtained from the searching program has the mapping as in Table V (a) on the next page, the four 2*D* constellations are shown in Fig. 10; its refined 16PSK version realizes mapping given in Table V (b). Simulation results are shown in Fig. 11.

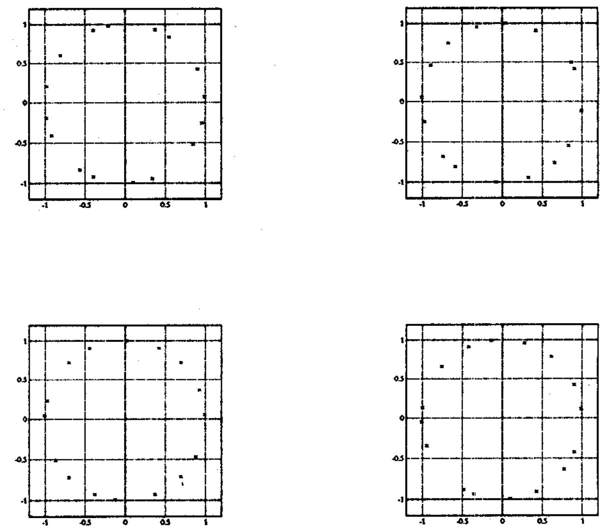

Fig. 10

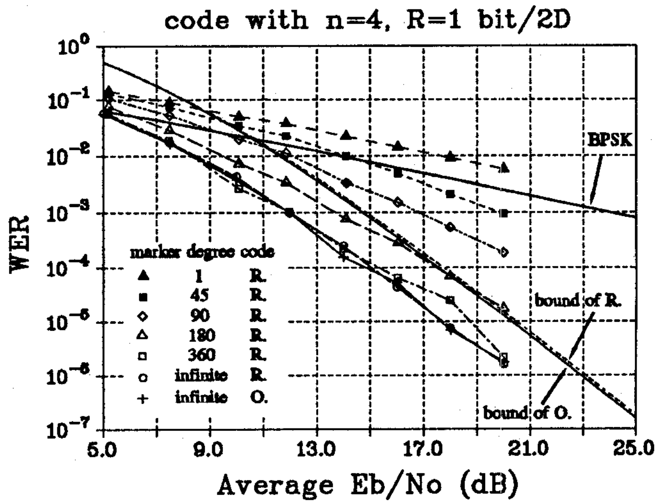

Fig. 11

From the simulation results, it can be seen that upon sufficient interleaving, all the codes provides significant gains in average SNR over the uncoded counterparts (if available). An interleaving degree more than 90 should be used to realize the gain, a degree between 180 and 360 is recommended. The loss caused by simplifying the original codes to a symmetric PSK codes is less than 0.2 dB for all the five cases.

Table IV (a): The mapping of the original (3,3) code

Code-words	Codeword coordinates		
	1st 2D	2nd 2D	3rd 2D
$s^{(0)}$	(-9.1317E-1,-4.0771E-1)	(4.9031E-2, 9.9899E-1)	(-9.3731E-1, 3.4887E-1)
$s^{(1)}$	(-3.8901E-1,-9.2130E-1)	(9.9450E-1,-1.0655E-1)	(3.9318E-1,-9.1961E-1)
$s^{(2)}$	(3.4463E-1,-9.3880E-1)	(-1.6370E-1,-9.8671E-1)	(3.9538E-1, 9.1866E-1)
$s^{(3)}$	(9.4219E-1,-3.3525E-1)	(-6.0818E-1, 7.9404E-1)	(9.1507E-1,-4.0363E-1)
$s^{(4)}$	(9.3566E-1, 3.5306E-1)	(6.5402E-1,-7.5673E-1)	(-9.4075E-1,-3.3950E-1)
$s^{(5)}$	(3.8901E-1, 9.2130E-1)	(-9.9450E-1, 1.0654E-1)	(-3.9318E-1, 9.1961E-1)
$s^{(6)}$	(-3.9948E-1, 9.1681E-1)	(7.9948E-1, 6.0102E-1)	(8.9541E-1, 4.4554E-1)
$s^{(7)}$	(-8.9734E-1, 4.4148E-1)	(-7.6258E-1,-6.4719E-1)	(-3.4042E-1,-9.4042E-1)

Table V (a): The mapping of the original (4,4) code

Code-words	Codeword coordinates			
	1st 2D	2nd 2D	3rd 2D	4th 2D
$s^{(0)}$	(9.062E-1, 4.228E-1)	(8.684E-1, 4.966E-1)	(2.464E-2, 1.000E+0)	(-4.842E-1,-8.753E-1)
$s^{(1)}$	(5.534E-1, 8.329E-1)	(3.322E-1,-9.436E-1)	(6.981E-1, 7.165E-1)	(6.210E-1, 7.842E-1)
$s^{(2)}$	(3.791E-1, 9.253E-1)	(-9.990E-1, 5.234E-2)	(-6.997E-1,-7.149E-1)	(-7.533E-1, 6.582E-1)
$s^{(3)}$	(-2.074E-1, 9.783E-1)	(-7.372E-1,-6.762E-1)	(7.019E-1,-7.127E-1)	(4.338E-1,-9.014E-1)
$s^{(4)}$	(-3.927E-1, 9.197E-1)	(4.302E-1, 9.031E-1)	(-9.994E-1, 4.267E-2)	(-9.403E-1,-3.412E-1)
$s^{(5)}$	(-8.030E-1, 5.959E-1)	(3.973E-2, 9.996E-1)	(9.297E-1, 3.692E-1)	(9.935E-1, 1.163E-1)
$s^{(6)}$	(-9.794E-1, 2.018E-1)	(6.564E-1,-7.549E-1)	(-6.993E-1, 7.153E-1)	(7.794E-1,-6.271E-1)
$s^{(7)}$	(-9.815E-1,-1.913E-1)	(9.092E-1, 4.171E-1)	(-3.729E-1,-9.282E-1)	(2.782E-1, 9.609E-1)
$s^{(8)}$	(-9.133E-1,-4.074E-1)	(-8.891E-1, 4.585E-1)	(4.232E-1, 9.064E-1)	(-9.994E-1,-4.233E-2)
$s^{(9)}$	(-5.583E-1,-8.296E-1)	(9.938E-1,-1.140E-1)	(9.990E-1, 5.120E-2)	(1.086E-1,-9.944E-1)
$s^{(10)}$	(-3.935E-1,-9.193E-1)	(-9.707E-1,-2.419E-1)	(-9.740E-1, 2.281E-1)	(9.075E-1, 4.209E-1)
$s^{(11)}$	(1.086E-1,-9.941E-1)	(-3.160E-1, 9.491E-1)	(8.842E-1,-4.679E-1)	(-4.187E-1, 9.085E-1)
$s^{(12)}$	(3.426E-1,-9.395E-1)	(-6.945E-2,-9.979E-1)	(-8.627E-1,-5.065E-1)	(-3.522E-1,-9.363E-1)
$s^{(13)}$	(8.551E-1,-5.184E-1)	(8.347E-1,-5.513E-1)	(3.818E-1,-9.246E-1)	(-9.909E-1, 1.367E-1)
$s^{(14)}$	(9.666E-1,-2.565E-1)	(-5.883E-1,-8.091E-1)	(-4.365E-1, 9.001E-1)	(-1.382E-1, 9.907E-1)
$s^{(15)}$	(9.975E-1, 7.082E-2)	(-6.699E-1, 7.429E-1)	(-1.195E-1,-9.932E-1)	(9.070E-1,-4.219E-1)

Table V (b):
Refined (4,4) code

Code-words	Phase number of codewords			
$s^{(0)}$	0	0	0	0
$s^{(1)}$	1	11	14	8
$s^{(2)}$	2	6	6	12
$s^{(3)}$	3	8	10	3
$s^{(4)}$	4	1	4	15
$s^{(5)}$	5	2	13	6
$s^{(6)}$	6	12	2	4
$s^{(7)}$	7	15	7	9
$s^{(8)}$	8	5	15	14
$s^{(9)}$	9	14	12	2
$s^{(10)}$	10	7	3	7
$s^{(11)}$	11	3	11	11
$s^{(12)}$	12	10	5	1
$s^{(13)}$	13	13	9	13
$s^{(14)}$	14	9	1	10
$s^{(15)}$	15	4	8	5

IV. Conclusions

The simulation confirmed that the full diversity PSK real number codes can provide significant coding gain when the symbols in a block are independently faded or when sufficient interleaving is used. The gain increases asymptotically with the block length, i.e., the diversity length at given coding rate, and also with the interleaving depth. The interleaving delay for these codes is low because of their short codeword lengths. For the codes considered in this paper, the loss in coding gain is negligible when the original codes were refined (or simplified) to symmetric PSK codes. The refined codes are simple for implementation.

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