Optimal Scheduling in High Speed Downlink Packet Access Networks (Online Supplement)

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We present an analytic model and a methodology to determine the optimal packet scheduling policy in a High Speed Downlink Packet Access (HSDPA) system. The optimal policy is the one that maximizes cell throughput while maintaining a level of fairness between the users in the cell. A discrete stochastic dynamic programming model for the HSDPA downlink scheduler is presented. Value iteration is then used to solve for the optimal scheduling policy. We use a FSMC (Finite State Markov Channel) to model the HSDPA downlink channel. A near optimal heuristic scheduling policy is developed. Simulation is used to study the performance of the resulted heuristic policy and compare it to the computed optimal policy.

Categories and Subject Descriptors: C.2.1 [**Computer Systems Organization**]: Computer Communication Networks, Network Architecture and Design—*wireless communication*; G.1.6 [**Mathematics of Computing**]: Numerical Analysis, Optimization—*Stochastic programming*; *unconstrained optimization*; G.1.10 [**Mathematics of Computing**]: Numerical Analysis—*applications*; G.3 [**Mathematics of Computing**]: Probability and Statistics—*stochastic processes*; *Markov processes*; I.6.3 [**Computing Methodologies**]: Simulation and Modeling—*applications, model development*

General Terms: Algorithms, Design, Performance, Theory

Additional Key Words and Phrases: Markov decision process, dynamic programming, optimal scheduling, resource allocation, HSDPA systems, 3G wireless networks, cross-layer design

1. SUPPLEMENTARY MATERIALS

In this section, we include supplementary graphs and results that we removed from the main text of the paper because of space limitation.

1.1 The additional sub-figures of Figure 5

Figures 4 and 5 in the main text describes the optimal policy structure for chunk size $c = 5$ and $c = 3$ respectively and for different arrival and channel quality parameters. However, in Figures 4 and 5 in the main text only the first two cases (case (a) and case (b)) were presented. In this supplement, we provide all the cases of Figure 4 (of the paper) in Figures 1–6. The cases of Figure 5 are shown in Figures 7–12 of this appendix.

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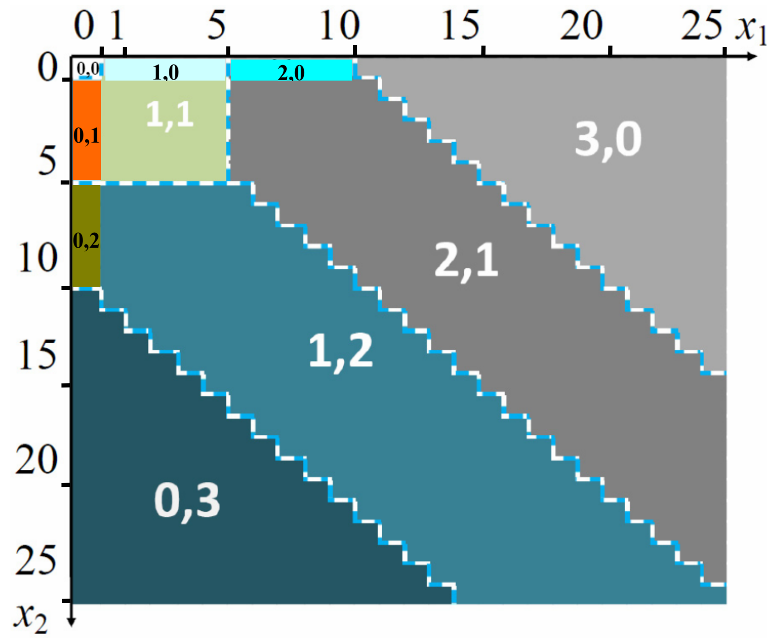

Fig. 1. Optimal and heuristic (dotted line) policies for two user case; $c = 5$ (i.e., 0,1,2 or 3 chunks of size 5 can be assigned to a user), $u = 5$ (Symmetrical case).

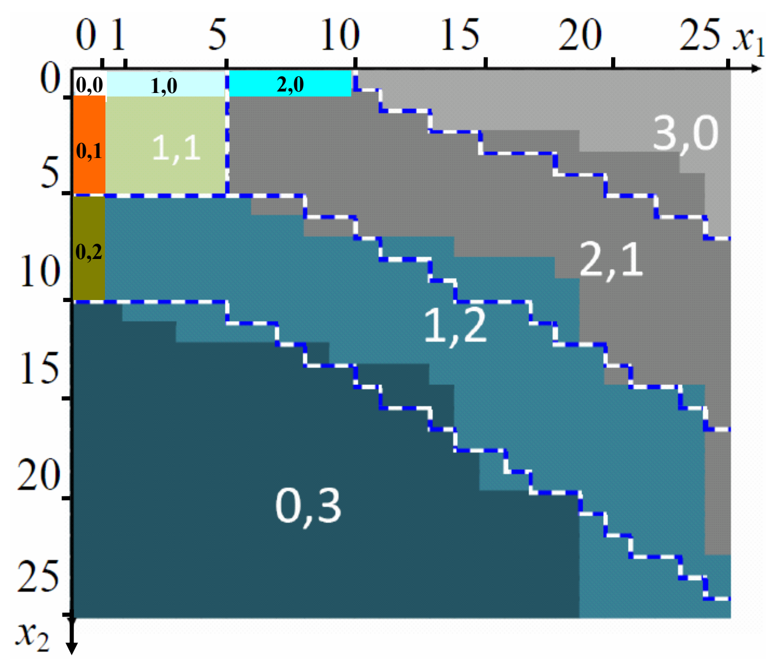

Fig. 2. Optimal and heuristic (dotted line) policies for two user case; $c = 5$ (i.e., 0,1,2 or 3 chunks of size 5 can be assigned to a user), $u = 5$ ($P(\gamma_1 = 1)$=0.8 and $P(\gamma_2 = 1)$=0.5).

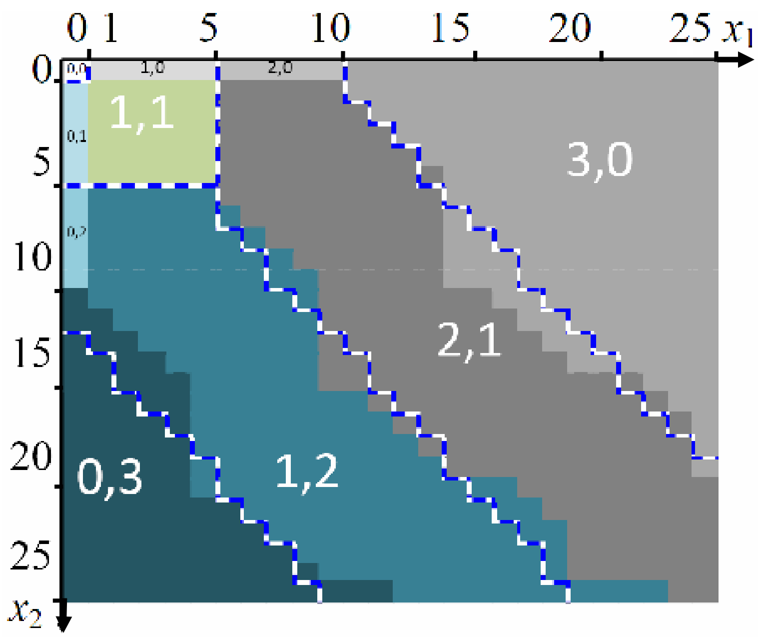

Fig. 3. Optimal and heuristic (dotted line) policies for two user case; $c = 5$ (i.e., 0,1,2 or 3 chunks of size 5 can be assigned to a user), $u = 5$ ($P(z_1 = 5) = 0.8$ and $P(z_2 = 5) = 0.5$).

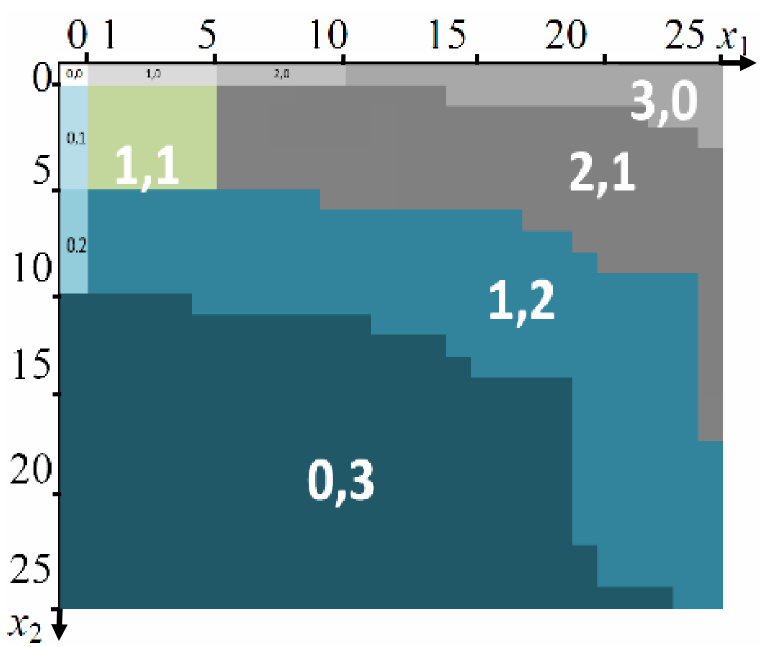

Fig. 4. Optimal and heuristic (dotted line) policies for two user case; $c = 5$ (i.e., 0,1,2 or 3 chunks of size 5 can be assigned to a user), $u = 5$ ($P(\gamma_1 = 1) = 0.8$ and $P(\gamma_2 = 1) = 0.3$).

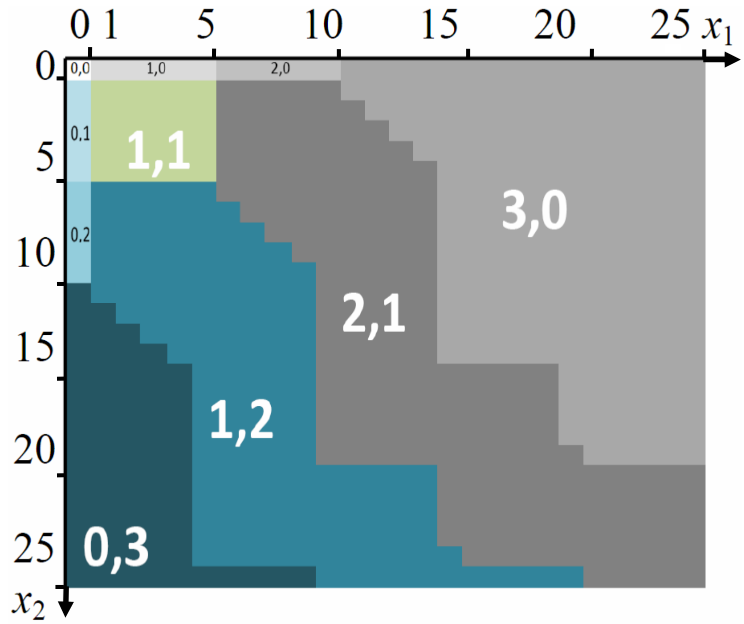

Fig. 5. Optimal and heuristic (dotted line) policies for two user case; $c = 5$ (i.e., 0,1,2 or 3 chunks of size 5 can be assigned to a user), $u = 5$ ($P(z_1 = 5)$=0.8 and $P(z_2 = 5)$=0.3).

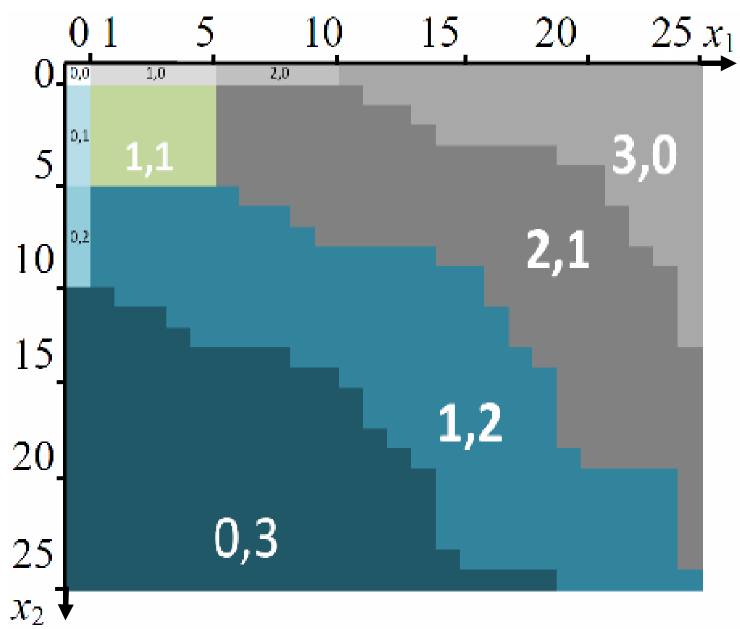

Fig. 6. Optimal and heuristic (dotted line) policies for two user case; $c = 5$ (i.e., 0,1,2 or 3 chunks of size 5 can be assigned to a user), $u = 5$ ($P(z_1 = 5) = 0.8$, $P(z_2 = 5) = 0.5$, $P(\gamma_1 = 1) = 0.8$ and $P(\gamma_2 = 1) = 0.5$).

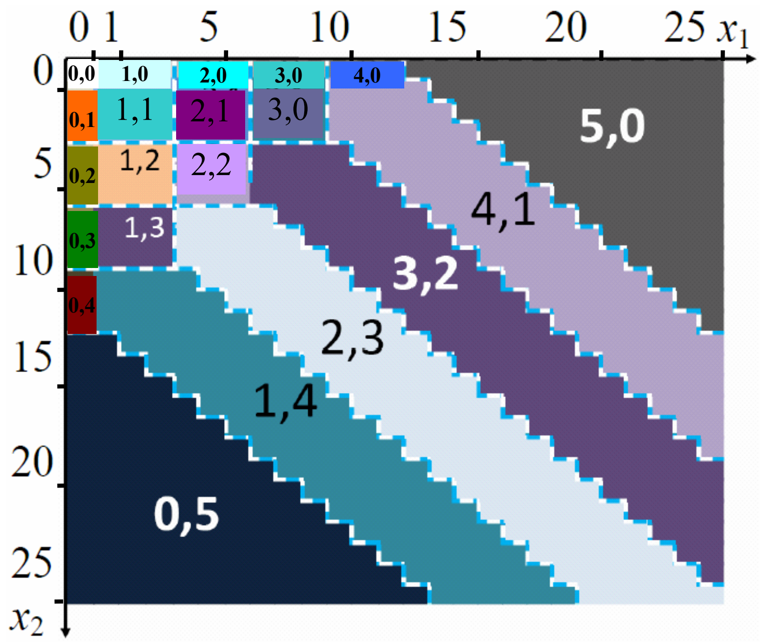

Fig. 7. Optimal and heuristic (dotted line) policies for two user case; $c = 3$ (i.e., 0,1,2,3,4 or 5 chunks of size 3 can be assigned to a user), $u = 5$ (Symmetrical case).

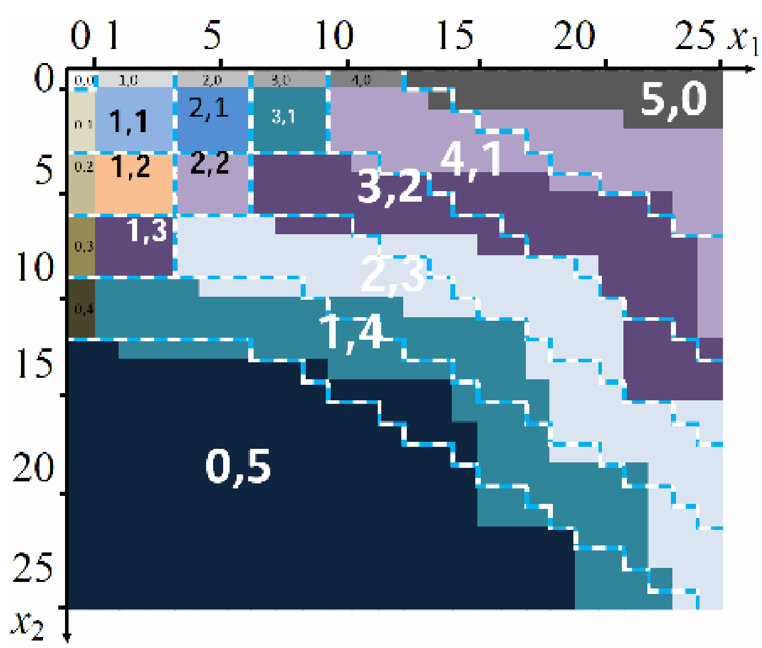

Fig. 8. Optimal and heuristic (dotted line) policies for two user case; $c = 3$ (i.e., 0,1,2,3,4 or 5 chunks of size 3 can be assigned to a user), $u = 5$ ($P(\gamma_1 = 1) = 0.8$ and $P(\gamma_2 = 1) = 0.5$).

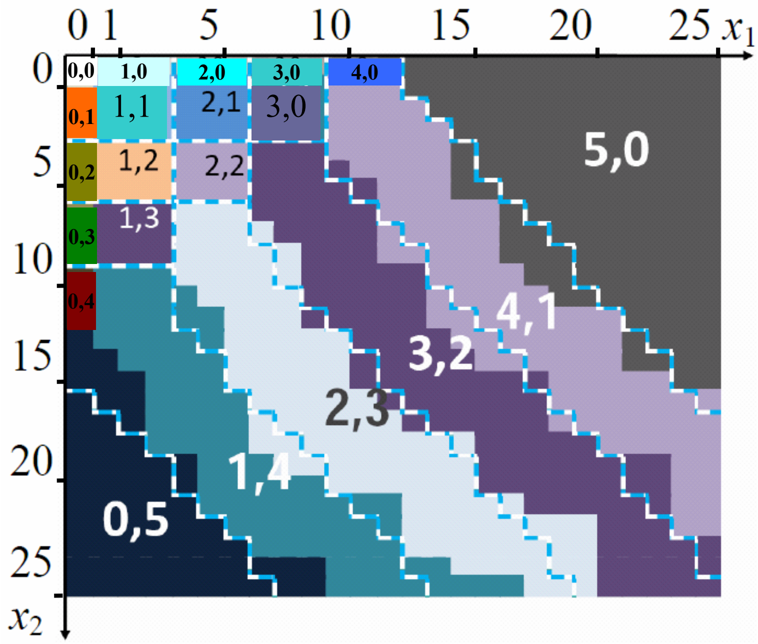

Fig. 9. Optimal and heuristic (dotted line) policies for two user case; $c = 3$ (i.e., 0,1,2,3,4 or 5 chunks of size 3 can be assigned to a user), $u = 5$ ($P(z_1=5)=0.8$ and $P(z_2=5)=0.5$).

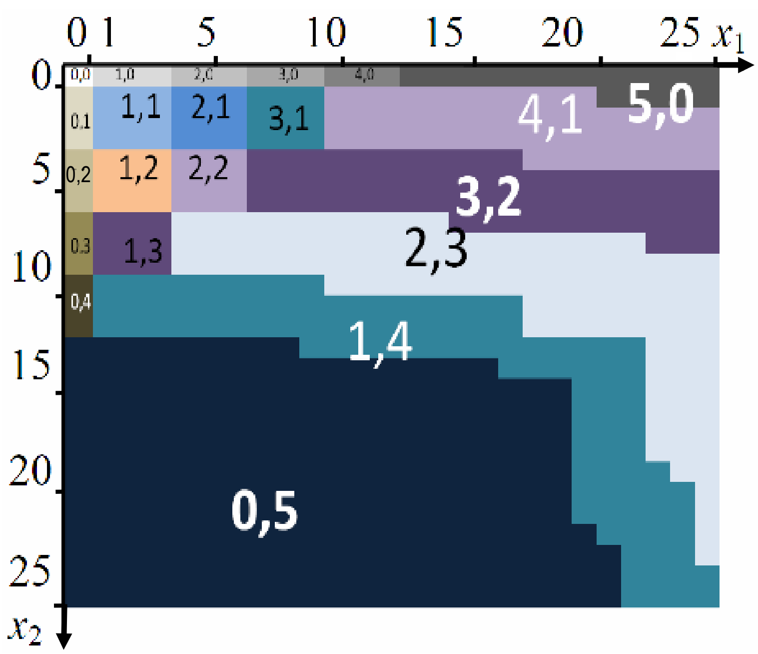

Fig. 10. Optimal and heuristic (dotted line) policies for two user case; $c = 3$ (i.e., 0,1,2,3,4 or 5 chunks of size 3 can be assigned to a user), $u = 5$ ($P(\gamma_1=1)=0.8$ and $P(\gamma_2=1)=0.3$).

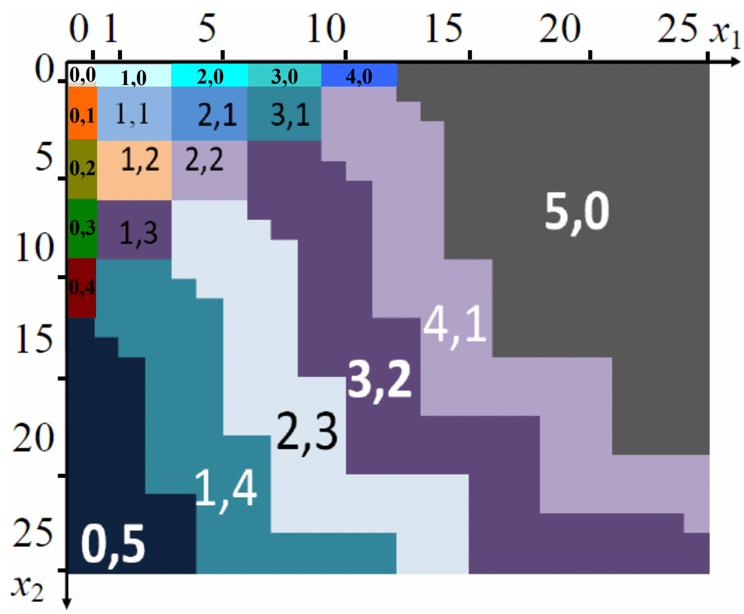

Fig. 11. Optimal and heuristic (dotted line) policies for two user case; $c = 3$ (i.e., 0,1,2,3,4 or 5 chunks of size 3 can be assigned to a user), $u = 5$ ($P(z_1 = 5) = 0.8$ and $P(z_2 = 5) = 0.3$).

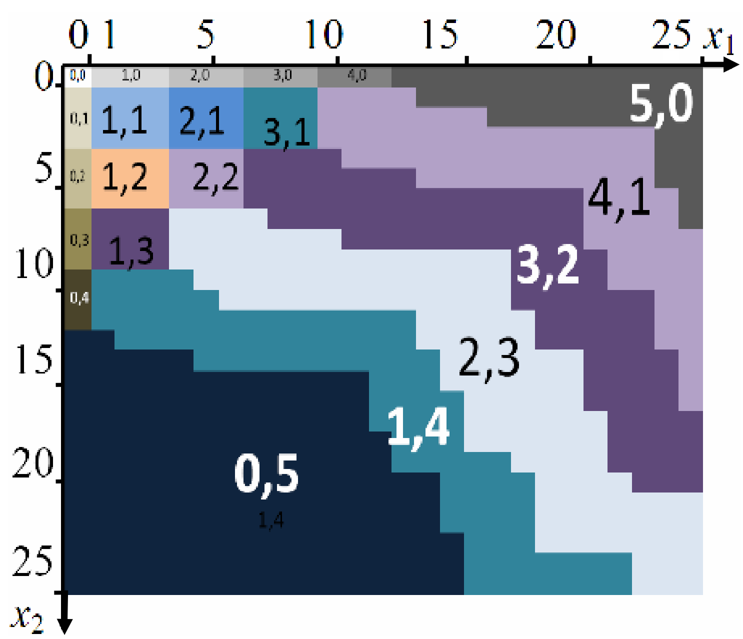

Fig. 12. Optimal and heuristic (dotted line) policies for two user case; $c = 3$ (i.e., 0,1,2,3,4 or 5 chunks of size 3 can be assigned to a user), $u = 5$ ($P(z_1 = 5) = 0.8$, $P(z_2 = 5) = 0.5$, $P(\gamma_1 = 1) = 0.8$ and $P(\gamma_2 = 1) = 0.5$).

SUPPLEMENTARY APPENDICES

This section contains all the appendices that were removed from the main text of the paper. We will use (*) to indicate references to materials in the main text of the paper to distinguish them from references in this supplement.

A. STATE TRANSITION PROBABILITY FOR *SECTION 3.5

To derive the state transition probability ($P_{\mathbf{ss'}}(\mathbf{a})$) that was introduced in *section 3, we start from *equation (4) as follows

$$\begin{aligned} P'_{\boldsymbol{ss}}(\boldsymbol{a}) &\triangleq Pr\big(\boldsymbol{s}(t+1) = \boldsymbol{s}' | \boldsymbol{s}(t) = \boldsymbol{s}, \boldsymbol{a}(t) = \boldsymbol{a}\big) \\ &= Pr(x'_1, \ldots, x'_L, \gamma'_1, \ldots, \gamma'_L | x_1, \ldots, x_L, \gamma_1, \ldots, \gamma_L, a_1, \ldots, a_L) \end{aligned} \quad \text{(A.1)}$$

where x_i denote the queue size of user i, and γ_i is the FSMC state characterizing the connectivity of user i wireless channel. Using conditioning we can decompose the above joint probability as follows:

$$\begin{aligned} P_{\boldsymbol{ss'}}(\boldsymbol{a}) &= Pr(x'_1, \ldots, x'_L | x_1, \ldots, x_L, \gamma_1, \ldots, \gamma_L, a_1, \ldots, a_L) \\ &\quad \cdot Pr(\gamma'_1, \ldots, \gamma'_L | x'_1, \ldots, x'_L, x_1, \ldots, x_L, \gamma_1, \ldots, \gamma_L, a_1, \ldots, a_L) \end{aligned} \quad \text{(A.2)}$$

Applying conditioning again, the second part of equation (A.2) yields

$$\begin{aligned} P_{\boldsymbol{ss'}}(\boldsymbol{a}) &= Pr(x'_1, \ldots, x'_L | x_1, \ldots, x_L, \gamma_1, \ldots, \gamma_L, a_1, \ldots, a_L) \\ &\quad \cdot Pr(\gamma'_1 | x'_1, \ldots, x'_L, x_1, \ldots, x_L, \gamma_1, \ldots, \gamma_L, a_1, \ldots, a_L) \\ &\quad \cdot Pr(\gamma'_2 | \gamma'_1, x'_1, \ldots, x'_L, x_1, \ldots, x_L, \gamma_1, \ldots, \gamma_L, a_1, \ldots, a_L) \cdot \ldots \\ &\quad \cdot Pr(\gamma'_L | \gamma'_1, \ldots, \gamma'_{L-1}, x'_1, \ldots, x'_L, x_1, \ldots, x_L, \gamma_1, \ldots, \gamma_L, a_1, \ldots, a_L) \end{aligned} \quad \text{(A.3)}$$

Since the wireless channel was modeled by means of a Markov process, the channel state γ_i depends only on the most recent channel state. Hence, the channel state transition probability can be written as:

$$Pr(\gamma'_i | \boldsymbol{s}) = Pr(\gamma'_i | \gamma_i) \triangleq P_{\gamma_i \gamma'_i}$$

Accordingly, we can rewrite (A.3) as follows:

$$P_{\boldsymbol{ss'}}(\boldsymbol{a}) = Pr(x'_1, \ldots, x'_L | x_1, \ldots, x_L, \gamma_1, \ldots, \gamma_L, a_1, \ldots, a_L) \cdot \prod_{i=1}^{L} P_{\gamma_i \gamma'_i} \quad \text{(A.4)}$$

Following the same approach, the joint probability of the queue size (first term in equation (A.2)) can be decomposed as follows:

$$\begin{aligned} & Pr(x'_1, \ldots, x'_L | x_1, \ldots, x_L, \gamma_1, \ldots, \gamma_L, a_1, \ldots, a_L) = \\ &\quad = Pr(x'_1 | x_1, \ldots, x_L, \gamma_1, \ldots, \gamma_L, a_1, \ldots, a_L) \\ &\quad \cdot Pr(x'_2 | x'_1, x_1, \ldots, x_L, \gamma_1, \ldots, \gamma_L, a_1, \ldots, a_L) \cdot \ldots \\ &\quad \cdot Pr(x'_L | x'_1, \ldots, x'_{L-1}, x_1, \ldots, x_L, \gamma_1, \ldots, \gamma_L, a_1, \ldots, a_L) \end{aligned} \quad \text{(A.5)}$$

The evolution of the queue size (x_i) is given by

$$\begin{aligned} x'_i &= \min\Big([x_i - y_i]^+ + z'_i \,, B\Big) \\ &= \min\Big([x_i - a_i \gamma_i c]^+ + z'_i \,, B\Big) \end{aligned} \quad \text{(A.6)}$$

Since the queue size corresponding to user i at the next time slot (x'_i) depends on its current queue size (x_i), the given action a_i, its channel state γ_i and the arrived PDUs z'_i during $(t, t+1]$ and is independent of all other queue sizes, actions and channel conditions corresponding to the remaining users. Hence

$$Pr(x'_i|\boldsymbol{s}) = Pr(x'_i|x_i, \gamma_i, a_i) \triangleq P_{x_i x'_i}(\gamma_i, a_i)$$

Therefore, equation (A.5) reduces to

$$P_{\boldsymbol{ss'}}(\boldsymbol{a}) = \prod_{i=1}^{L} \left(P_{x_i x'_i}(\gamma_i, a_i)\, P_{\gamma_i \gamma'_i} \right) \tag{A.7}$$

The state transition probability will be the product of the individual user queues state transition probabilities and their FSMC channel transition probabilities. The underlining assumption is that $P_{\gamma_i \gamma'_i}$ can in practice be estimated from measurements and provided to the scheduler. The term $P_{x_i x'_i}(\gamma_i, a_i)$ will be derived in the following section.

B. QUEUE STATE TRANSITION PROBABILITY FOR *SECTION 3.5

The queue transition probability is given by

$$P_{x_i x'_i}(\gamma_i, a_i) = Pr\big(x_i(t+1) = x'_i | x_i(t) = x_i, \gamma_i(t) = \gamma_i, a_i(t) = a_i\big) \tag{B.1}$$

where

$$x_i(t+1) = \min\Big([x_i(t) - y_i(t)]^+ \; + z_i(t+1)\,, B \Big) \tag{B.2}$$

with $y_i(t) = a_i(t)\gamma_i(t)c$. From equation (B.2) we can differentiate two cases; (a) $[x_i(t) - y_i(t)]^+ \; + z_i(t+1) \; < B$ and (b) $[x_i(t) - y_i(t)]^+ \; + z_i(t+1) \; \geq B$. The objective of this section is to derive $P_{x_i x'_i}(\gamma_i, a_i)$ for both cases. Substituting the two cases in equation (B.1) yields the following

Case (a): $[x_i(t) - y_i(t)]^+ \; + z_i(t+1) \; < B$ or equivalently

$$x_i(t+1) = [x_i(t) - y_i(t)]^+ \; + z_i(t+1)$$

Equation (B.1) in this case can be rewritten as:

$$\begin{aligned} P_{x_i x'_i}(\gamma_i, a_i) &= Pr\big([x_i(t) - y_i(t)]^+ + z_i(t+1) = x'_i \; | x_i(t) = x_i, \gamma_i(t) = \gamma_i, a_i(t) = a_i \big) \\ &= Pr\big([x_i - y_i]^+ \; + z_i(t+1) = x'_i \big) \\ &= Pr\big(z_i(t+1) = x'_i - [x_i - y_i]^+ \big) \end{aligned} \tag{B.3}$$

where $y_i = a_i \gamma_i c$. The arrival process is assumed to be Bernoulli with parameter q_i for user i (*section 3.1) and is given by

$$z_i(t) = \begin{cases} u_i & \text{with probability } q_i \\ 0 & \text{with probability } 1 - q_i \end{cases} \tag{B.4}$$

Hence, the queue state transition probability in this case is

$$P_{x_i x'_i}(\gamma_i, a_i) = \begin{cases} q_i & \text{if } x'_i = [x_i - y_i]^+ + u_i \\ 1 - q_i & \text{if } x'_i = [x_i - y_i]^+ \\ 0 & \text{Otherwise} \end{cases} \tag{B.5}$$

Case (b): $[x_i(t) - y_i(t)]^+ + z_i(t+1) \geq B$ or equivalently $x_i(t+1) = B$. In this case, one can conclude that $P_{x_i x_i'}(\gamma_i, a_i) = 0$ when $x_i' \neq B$. The remaining of this section is devoted to the calculation of $P_{x_i B}(\gamma_i, a_i)$ as follows:

$$\begin{aligned} P_{x_i B}(\gamma_i, a_i) &= Pr\big([x_i(t) - y_i(t)]^+ + z_i(t+1) \geq B \,|x_i(t) = x_i, \gamma_i(t) = \gamma_i, a_i(t) = a_i\big) \\ &= Pr\big([x_i - y_i]^+ + z_i(t+1) \geq B\big) \\ &= Pr(z_i(t+1) \geq B - [x_i - y_i]^+) \end{aligned} \tag{B.6}$$

Similar to case (a), we can write equation (B.6) as follows

$$P_{x_i B}(\gamma_i, a_i) = \begin{cases} 1 - q_i & \text{if } [x_i - y_i]^+ \geq B \\ q_i & \text{if } [x_i - y_i]^+ + u_i \geq B \\ 0 & \text{Otherwise} \end{cases} \tag{B.7}$$

Using our knowledge of queue evolution process, we can summarize all the possible transitions in (B.7) by partitioning the probability space of $P_{x_i B}(\gamma_i, a_i)$ as follows:

$$P_{x_i B}(\gamma_i, a_i) = \begin{cases} 1 - q_i & \text{if } x_i = B,\ \gamma_i a_i = 0 \\ q_i & \text{if } x_i = B,\ \gamma_i a_i = 0 \\ q_i & \text{if } x_i = B,\ 0 < \gamma_i a_i c \leq u_i \\ q_i & \text{if } x_i < B,\ [x_i - \gamma_i a_i c]^+ + u_i \geq B \\ 0 & \text{Otherwise} \end{cases} \tag{B.8}$$

where the first case in equation (B.8) (corresponding to probability $1 - q_i$) is the only possible conditions for the first case in equation (B.7). The other three cases (corresponding to probability q_i) represent all the possible partitions of the probability space of the second case in equation (B.7).

The results obtained above in (B.5) and (B.8) can be summarized as follows

$$P_{x_i x_i'}(\gamma_i, a_i) = \begin{cases} 1 & \text{if } x_i' = x_i = B,\ \gamma_i a_i = 0 \\ q_i & \text{if } x_i' = x_i = B,\ 0 < \gamma_i a_i c \leq u_i \\ q_i & \text{if } x_i' = B,\ x_i < B,\ [x_i - \gamma_i a_i c]^+ + u_i \geq B \\ q_i & \text{if } x_i' < B,\ x_i' = [x_i - \gamma_i a_i c]^+ + u_i \\ 1 - q_i & \text{if } x_i' < B,\ x_i' = [x_i - \gamma_i a_i c]^+ \\ 0 & \text{Otherwise} \end{cases} \tag{B.9}$$

C. DERIVATION OF THE QUEUE STATE TRANSITION PROBABILITY IN *SECTION 5.3

In this section, we present the derivation of queue state transition probabilities when retransmission is considered (*section 5.3). There are two parts for this derivation; (a) when the transmission is successful (*equation (15)) and (b) when the transmission is unsuccessful (*equation (16)). In case (a), the conditional probability $P_{x_i x_i' | \mu_i = 1}(\gamma_i, a_i)$ is similar to that in equation (B.1) and the derivation is analogous to that in the previous section and will not be repeated here.

The remainder of this section is devoted to derive the probability in case (b); namely, the queue transition probability when the transmission is unsuccessful (i.e.,

$\mu_i(t) = 0$). In this case, no PDUs are removed from the scheduled queues at the end of the current TTI (since the transmission was unsuccessful). The conditional queue state transition probability is given by

$$P_{x_i x_i'|\mu_i=0}(\gamma_i, a_i) \triangleq Pr\big(x_i(t+1) = x_i' | x_i(t) = x_i, \gamma_i(t) = \gamma_i, a_i(t) = a_i, \mu_i = 0\big) \quad \text{(C.1)}$$

where $x_i(t+1)$ is given by *equation (12). Similar to section B, we can distinguish the following two cases:

case (b1): $[x_i(t) - y_i(t)\mu_i(t)]^+ + z_i(t+1) < B$; equivalently,

$$x_i(t+1) = [x_i(t) - y_i(t)\mu_i(t)]^+ + z_i(t+1)$$

Equation C.1 can be rewritten as:

$$\begin{aligned} P_{x_i x_i'|\mu_i=0}(\gamma_i, a_i) &= Pr\big([x_i(t) - y_i(t)\mu_i(t)]^+ + z_i(t+1) = x_i' \,| x_i(t) = x_i, \\ &\quad \gamma_i(t) = \gamma_i, a_i(t) = a_i, \mu_i = 0\big) \\ &= Pr(x_i + z_i(t+1) = x_i') \\ &= Pr(z_i(t+1) = x_i' - x_i) \qquad \text{(C.2)} \end{aligned}$$

As mentioned previously, $z_i(t)$ has Bernoulli distribution with parameter q_i. Therefore, the marginal queue state transition probability is given by

$$P_{x_i x_i'|\mu_i=0}(\gamma_i, a_i) = \begin{cases} q_i & \text{if } x_i' = x_i + u_i \\ 1 - q_i & \text{if } x_i' = x_i \\ 0 & \text{Otherwise} \end{cases} \quad \text{(C.3)}$$

case (b2): $[x_i(t) - y_i(t)\mu_i(t)]^+ + z_i(t+1) \geq B$ equivalently $x_i(t+1) = B$, then equation C.1 can be rewritten as

$$\begin{aligned} P_{x_i B|\mu_i=0}(\gamma_i, a_i) &= Pr([x_i(t) - y_i(t)\mu_i(t)]^+ + z_i(t+1) \geq B \,| x_i(t) = x_i, \\ &\quad \gamma_i(t) = \gamma_i, a_i(t) = a_i, \mu_i = 0) \\ &= Pr(x_i + z_i(t+1) \geq B) \\ &= Pr(z_i(t+1) \geq B - x_i) \qquad \text{(C.4)} \end{aligned}$$

and $P_{x_i x_i'|\mu_i=0}(\gamma_i, a_i) = 0$ when $x_i' \neq B$.
Using equation (B.4), we can conclude that

$$P_{x_i B|\mu_i=0}(\gamma_i, a_i) = \begin{cases} q_i & \text{if } x_i + u_i \geq B \text{ and } x_i < B \\ 1 & \text{if } x_i = B \\ 0 & \text{Otherwise} \end{cases} \quad \text{(C.5)}$$

The second case in equation C.5 is the aggregation of two events (arrival and no arrival), since both events will result in $x_i' = B$ when $x_i = B$, i.e., if the queue is full initially and no PDUs are removed from this queue in the current TTI (due to unsuccessful transmission), then the queue will remain full regardless of the arrival status.

Combining equations (C.3) and (C.5) yields

$$P_{x_i x_i'|\mu_i=0}(\gamma_i, a_i) = \begin{cases} 1 & \text{if } x_i' = x_i = B \\ q_i & \text{if } x_i' = B, x_i < B, x_i + u_i \geq B \\ q_i & \text{if } x_i' < B, x_i' = x_i + u_i \\ 1 - q_i & \text{if } x_i' < B, x_i' = x_i \\ 0 & \text{Otherwise} \end{cases} \tag{C.6}$$