Downlink Retainability of LEO Satellite to Internet of Remote Things

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***Abstract*: Some ground devices in Internet of remote things are expected to directly access to low Earth orbit satellites. In this paper, we evaluate the communication retainability for a generic IoRT cluster that consists of one branch from the satellite and the other from the unmanned aerial vehicle. The fading model is based on the Hoyt distributions. The closed-forms of communication retainability of the composite Hoyt-Rayleigh fading of maximal-ratio combining and selection combining are derived. Profile examples are illustrated to show the effects of several main parameters. It is observed that MRC results in the higher communication retainability than SC.**

Index Terms: Internet of remote things (IoRT), Nakagami-q distribution, satellite communication, signal fading.

I. Introduction

The *Beyond-5G* (B5G) wireless technologies should minimize, or ideally, eliminate the "no-signal" problem due to the lack of coverage by conventional cellular networks. Currently, the interest is high in supporting the communications between the *low Earth orbit* (LEO) satellites and user devices such as smartphones and devices in *Internet of remote things* (IoRT) ([1], [2]). The altitude of a LEO satellite is typically in the range from 160 km to 1600 km, sometimes up to 2000 km (In contrast, the GPS satellites are in *medium Earth orbit* at the altitude of about 20200 km). Since the ionosphere is from about 48 km to 965 km altitude, a big portion of the transmission path between the LEO satellites and ground devices is usually impacted by ionospheric scintillations. Thus, satellite-smartphone communication may also be complemented by some intermediary flying nodes such as *unmanned aerial vehicles* (UAVs). This results in a hybrid system. A generic view is illustrated in Fig. 1.

The reliability of satellite communication is of paramount importance and has received a great deal of attention since the early era of space technologies. For *geostationary* (GEO) satellites, the *link availability* is used as a measure to describe the percentage of time in which the communication link is closed. The link availability is a notion on the average over a specified long term [3, Ch. 5]. For LEO satellites, we introduce a measure, called the *communication retainability*, to describe the short-term behavior of communication channels. In this paper, we will derive the closed-form expression of the communication retainability for LEO satellite-ground communications aided by UAVs. Since it is well known for the pros and cons of Monte Carlo simulations, we expect the analytical results to provide some decent references and/or reasonable benchmarks for simulations if necessary.

On the basis of analytical results, we will compare the performance of *maximal-ratio combining* (MRC) with *selection combining* (SC). This is mainly because SC is the simplest combining scheme of linear diversity combining methods, and it is worth knowing the relative merit of other combining schemes such as MRC. If the performance of SC is above some thresholds, then the less complicated structure of SC would be a feasible solution for most cellular hand-held and compact devices in IoRT. As shown in this paper, the derivation of communication retainability for MRC involves several advanced special functions. The details have not been found elsewhere.

The rest of this paper is organized as follows. In Sec. II, the system essentials are described. Then some relevant works are reviewed in Sec. III. Next, in Sec. IV, the closed-forms of communication retainability of MRC and SC are presented. The derivation is included in the Appendix. Then, in Sec. V, several numerical profiles are provided and discussed. Finally, the conclusion is included in Sec. VI.

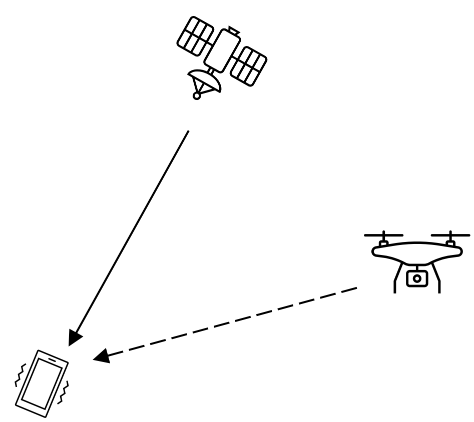

Figure 1. Generic system model.

II. System Essentials

The Hoyt distribution [4] is one of the conventional statistical models in telecommunications. It is also referred to as the *Nakagami-q* distribution ([5], [6]). In the context of satellite communications, the Hoyt distribution has been applied to describe the small-scale fading[1] induced by strong ionospheric scintillation [7]. Note that the ionosphere is from about 48 km to 965 km altitude, which is a significant portion of the transmission path between the LEO satellites and ground devices in IoRT.

In this paper, the analysis is conducted for a generic system shown in Fig. 1. The present work focuses on the downlink transmissions, since the fading statistics of uplink transmissions are different, and it is reasonable to make the corresponding investigation in another paper. In the downlink transmissions, the *envelope* of signals transmitted from the satellite to the user device is impacted by the *Hoyt* fading, while it is subject to the *Rayleigh* fading from UAV to use device. The instantaneous *signal-to-noise ratio* (SNR) per signal symbol will be denoted as U and W, for the satellite channel and UAV channel, respectively. Note that both U and W are *random variables* (RVs) under the usual condition of flat fading. For the concerned scenario involving the satellite-ground channel and the UAV-ground channel, it is assumed that U and W are statistically independent.

For practical LEO-satellite-ground communications in IoRT, more issues need to be addressed. For example, unlike GEO satellites, LEO satellites move quickly relative to the ground devices. The *Doppler shift* needs to be considered in the link budget analysis. Moreover, the large-scale fading is dependent on the carrier frequency. These issues are addressed in Sec. V where some numerical experiments are presented.

III. BACKGROUND AND RELEVANT STUDIES

Originally, the Hoyt variety stemmed from the communication system where the inphase and quadrature Gaussian components are statistically independent and have non-identical variances with zero means. It can also be equivalently applied to the case where the inphase and quadrature Gaussian components are statistically correlated and have identical variances with zero means. Mathematically, the *probability density function* (PDF) of the squared Hoyt variety U can be expressed in several forms, which are all equivalent to each other [4-6]. Here we adopt the formulation presented in [6, eq. (2.11)]:

$$f_U(u) = \frac{1+p^2}{2pu_0}\exp\left[-\frac{(1+p^2)^2 u}{4p^2 u_0}\right] I_0\left[\frac{(1-p^4)u}{4p^2 u_0}\right]. \qquad (u \ge 0) \tag{1}$$

In (1), $I_0(\bullet)$ is the *modified Bessel function of the first kind of order zero*, p is the Hoyt fading parameter $(0 < p \le 1)$, and u_0 is the average of U. The *cumulative distribution function* (CDF) of U is not provided in [6] but there was a derivation in [9]:

$$F_U(u) = 1 - 2Q\left[\frac{(1-p)}{2p}\sqrt{\frac{(1+p^2)u}{u_0}}, \frac{(1+p)}{2p}\sqrt{\frac{(1+p^2)u}{u_0}}\right] + \exp\left(-\frac{(1+p^2)^2 u}{4p^2 u_0}\right) I_0\left[\frac{(1-p^4)u}{4p^2 u_0}\right], \tag{2}$$

where $Q(\bullet,\bullet)$ is the first-order *Marcum Q-function*. Note that, in (1) and (2), u_0 represents the average SNR corresponding to channel gain owing to alone the shadowing effect and large-scale fading [8, pp. 70-71]. The large-scale fading mainly depends on the distance and the path-loss coefficient. A further discission of u_0 is presented in Sec. V.

It is worth mentioning that, in the literature, the terms of "Hoyt RV" and "squared Hoyt RV" are sometimes interchangeably used. This usage would be acceptable if the study was concerned with the single RV, the product of multiple RVs, or the ratio of two RVs. This is because for the ratio and product of fading varieties, the distribution functions of the Hoyt RV are equivalent to those of the squared Hoyt RVs with an elementary transform. However, for the sum of two RVs, we need to distinguish the sum of squared Hoyt RVs from the sum of Hoyt RVs. In communication systems, the *equal gain combining* (EGC) scheme is based on the sum of RVs, whereas the MRC scheme corresponds to the sum of squared RVs. It is well known that the SNR or the power metric is associated to the squared RVs. Throughout this paper, we explicitly state the prefix "the squared" whenever necessary.

Compared with other fading models like *Nakagami-m*, *Nakagami-n* (Rice), and *Weibull* distributions, it is more challenging to seek the exact and closed-form for the distribution functions of composite Hoyt models. Sometimes more sophisticated schemes are needed to deal with the CDF than PDF. Studies for the single Hoyt RVs can be found in [10] and [11], related to optical wireless communications. Beyond that, the ratio of two non-identically distributed squared-Hoyt was studied in [9] and an exact formula of the CDF was derived. Later this result was simplified for the scenarios where one of the channels is with the unit Hoyt factor and more compact formulations were obtained [12]. On the other hand, the product of Hoyt RVs has also been well investigated in several terrestrial communications (see [13] and the references therein). With regard to the sum of Hoyt RVs, an approximation approach was proposed in [14] by means of the moment-match scheme. Later, a study on the sum of squared Hoyt RVs was presented in [15]. However, the results obtained in [15] were excessively complicated: the PDF was expressed in terms of an infinite series and the CDF involved triple nested series, so the truncation errors must be carefully estimated in performance evaluation. It should be mentioned that there have also been some continuous interests in pursuing approximate approaches or numerical schemes. The review for this area is omitted here since our

[1] Fading refers to the rapid variation of the amplitude and phase of a received signal in trans-ionosphere propagation.

main interest is in the exact and closed-form expressions. For the present paradigm, it is desirable to seek a compact solution without infinite series. In the next section, we develop the analysis along this avenue. Our final results do not involve integrals nor infinite series.

IV. Composite Probability Distribution of Hoyt and Rayleigh Random Variables

As described in Sec. II, the current paradigm involves a fading channel near ground, between UAV and ground users. For this channel, the signal envelope is impacted by the Rayleigh fading and the PDF of received SNR W is exponential [6, eq. (2.7)]:

$$f_W(w) = \frac{1}{w_0}\exp\left(-\frac{w}{w_0}\right), \quad (w \ge 0) \tag{3}$$

where w_0 is the average of W, with an interpretation similar to u_0. Note that the pathloss exponent may differ from that of u_0. According to (3), the CDF of W is:

$$F_W(w) = 1 - \exp\left(-\frac{w}{w_0}\right), \tag{4}$$

Let $Z = U + W$ be the SNR of maximal-ratio combining (MRC). Then the CDF of Z can be expressed as follows:

$$\begin{aligned} F_Z(z) &= \int_0^z F_W(z-u) f_U(u)du \\ &= F_U(z) - \exp\left(-\frac{z}{w_0}\right)\int_0^z \exp\left(\frac{u}{w_0}\right) f_U(u)du \\ &= F_U(z) - \frac{1+p^2}{2pu_0}\exp\left(-\frac{z}{w_0}\right) J_z, \end{aligned} \tag{5}$$

where

$$J_z = \int_0^z \exp\left\{-\left[\frac{(1+p^2)^2}{4p^2u_0} - \frac{1}{w_0}\right]u\right\} I_0\left[\frac{(1-p^4)u}{4p^2u_0}\right]du. \tag{6}$$

Moreover, the corresponding PDF is:

$$f_Z(z) = \frac{1}{w_0}\exp\left(-\frac{z}{w_0}\right)\int_0^z \exp\left(\frac{u}{w_0}\right) f_U(u)du. \tag{7}$$

Comparing (7) with (5), we obtain

$$F_Z(z) = F_U(z) - w_0 f_Z(z). \tag{8}$$

Note that the closed-form of the first term $F_U(\bullet)$ in (5) is already available in (2), so the main task is to solve (6). The solving strategy of the integral J_z is fairly long and provided in the Appendix. As a result, with a reasonable condition, we reach the following exact and closed-form expression:

$$J_z = \frac{2pu_0w_0}{\sqrt{[w_0(1+p^2) - 2p^2u_0][w_0(1+p^2) - 2u_0]}} \times \left\{1 - 2Q(a,b) + \exp\left(\frac{z}{w_0} - \frac{(1+p^2)^2 z}{4p^2u_0}\right) I_0\left[\frac{(1-p^4)z}{4p^2u_0}\right]\right\}. \tag{9}$$

Finally, substituting (2) and (9) into (5), we obtain:

$$\begin{aligned} F_Z(z) &= 1 - 2Q\left[\frac{(1-p)}{2p}\sqrt{\frac{(1+p^2)z}{u_0}}, \frac{(1+p)}{2p}\sqrt{\frac{(1+p^2)z}{u_0}}\right] \\ &\quad + \exp\left(-\frac{(1+p^2)^2 z}{4p^2u_0}\right) I_0\left[\frac{(1-p^4)z}{4p^2u_0}\right] \\ &\quad - \frac{(1+p^2)w_0}{\sqrt{[w_0(1+p^2) - 2p^2u_0][w_0(1+p^2) - 2u_0]}} \\ &\quad \times \left\{[1 - 2Q(a,b)]\exp\left(-\frac{z}{w_0}\right)\right. \\ &\quad \left. + \exp\left[-\frac{(1+p^2)^2 z}{4p^2u_0}\right] I_0\left[\frac{(1-p^4)z}{4p^2u_0}\right]\right\}. \end{aligned} \tag{10}$$

Due to (10), the PDF of Z takes the following form:

$$\begin{aligned} f_Z(z) &= \frac{1+p^2}{\sqrt{[(1+p^2)w_0 - 2p^2u_0][(1+p^2)w_0 - 2u_0]}} \\ &\quad \times \left\{\left[1 - 2Q(a,b)\right]\exp\left(-\frac{z}{w_0}\right)\right. \\ &\quad \left. + \exp\left[-\frac{(1+p^2)^2 z}{4p^2u_0}\right] I_0\left[\frac{(1-p^4)z}{4p^2u_0}\right]\right\}. \end{aligned} \tag{11}$$

In the following, we shift the investigation to the SC scheme, recalling that SC represents the least complicated scheme of linear diversity combining methods [6].

Let $T = \max(U, W)$ be the SNR of SC. Then the CDF of T can be expressed as follows:

$$\begin{aligned} F_T(t) &= \Pr[\max(U,W) < t] \\ &= \Pr(U < t)\Pr(W < t) = F_U(t)F_W(t). \end{aligned} \tag{12}$$

As shown in (12), unlike MRC, the CDF of SC has a simple form as the product of two single CDFs. Nevertheless, due to (8), an analytical relation between the fading distribution functions of MRC and SC can be conveniently expressed as:

$$F_T(t) = F_U(t)F_W(t) = [F_Z(t) + w_0 f_Z(t)]F_W(t). \tag{13}$$

The outage probability is a common metric in wireless communication over fading channels. Its original notion stemmed from the probability that a given information rate is not supported. It can be equivalently expressed in terms of SNRs if some parameters are fixed. In LEO satellite communications, the notion of outage probability can be conveniently adopted to describe the *communication retainability*. For the MRC and SC schemes, the communication retainability is respectively defined as

$$p_{MRC} = \Pr(Z > \gamma_{th}) = 1 - F_Z(\gamma_{th}), \tag{14}$$

$$p_{SC} = \Pr(T > \gamma_{th}) = 1 - F_T(\gamma_{th}), \tag{15}$$

where γ_{th} is the prespecified threshold. It can be shown that

$$F_Z(t) < F_T(t). \tag{16}$$

In other words, MRC has a higher communication retainability than SC for the same threshold. However, it is desirable to know some representative quantitative profiles, so SC could still be used within a tolerable range. This is important since SC is a simpler scheme than MRC and would be more suitable to those compact devices. Some quantitative profiles are shown in Sec. V.

V. Numerical Profiles and Remarks

Multiple bands of spectra are allowed in the 5G-compatible satellite communication systems, e.g., the S band (2 to 4 GHz) and the Ka band (20 GHz for downlink, 30 GHz for uplink) ([1], [2]). As one of the main design parameters for satellite transmitters, the *effective isotropic radiated power* (EIRP) density is spectrum dependent. The main parameters are listed in Table I.

TABLE I. Main Parameters

Parameter	Value
LEO Satellite altitude	600 km
Satellite EIRP density (S Band)	34 dBW/MHz
Downlink carrier freq. (S Band)	2 GHz
Channel bandwidth (S Band)	30 MHz
Subcarrier spacing (S Band)	15 kHz
Satellite EIRP density (Ka Band)	4 dBW/MHz
Downlink carrier freq. (Ka Band)	20 GHz
Channel bandwidth (Ka Band)	400 MHz
Subcarrier spacing (Ka Band)	60 kHz
Satellite pathloss exponent	2
UAV altitude	100 to 500 m
UAV transmit power	0.5 W
UAV pathloss exponent	2 to 4

In addition to the parameters in Table I, if the *effective system noise temperature* (ESNT) T° and receiver antenna gain G_r or their ratio *G/T* are further given, it is possible to estimate the downlink SNR. According to the standard treatment for the link budget in satellite-ground communications [3, Ch. 5], the downlink SNR can be expressed as follows:

$$SNR = \left(\frac{EIRP}{L_s B\kappa}\right) \times \left(\frac{G_r}{T^\circ}\right), \quad (17)$$

where L_s is the overall fading losses due to atmosphere, scintillation, and propagation distance, B is the user bandwidth, and κ is the Boltzmann's constant. For example, in the case of Ka Band, G_r = 39.7 (dBi) and $L_s \approx$ 174.8 (dB). Moreover, in practice, some appropriate compensation schemes for Doppler shift may be introduced. This is the LEO satellites keep moving relative to the ground. The Doppler frequency could be estimated with:

$$f_{Doppler} = v_s f_c / c, \quad (18)$$

where c, f_c, and v_s are the speed of light, carrier frequency, and the relative speed of satellite to ground, respectively. For example, with $v_s \approx 7.6$ km/sec., we would have $f_{Doppler} \approx 509$ (kHz) and 50.9 (kHz) for the downlink communications of the Ka Band and S Band, respectively. Recently, several techniques to compensate the Doppler shift effects have been well developed, in particular for downlink signals (see [16], [17], and the references therein). To focus on the main insights, in numerical experiments, the Doppler effect can be either implicitly included in the modified carrier frequency or explicitly introduced to the range equation such as (17). The former is adopted in the present experiments.

As introduced in (1), the parameter u_0 is a key parameter of the LEO-ground channel. In practice, u_0 is usually evaluated after the device gain is applied. This is because, in wireless communications, the device gain should well compensate for overall attenuations due to pathloss and shadowing. Accordingly, u_0 can be estimated through (17). On the other hand, the parameter w_0 of the UAV-ground channel can be estimated through the *ray-tracing* model with pathloss exponent $\alpha_w = 2 \sim 4$ [8, Sec. 2.4].

Some example profiles are illustrated in Figs. 2 through 5, where the communication retainability threshold is labelled in the abscissa. As shown in Fig. 2, the communication retainability of MRC is much higher than SC. This merit gap becomes wider when the mean of UAV-ground channel becomes larger, as shown in Fig. 3.

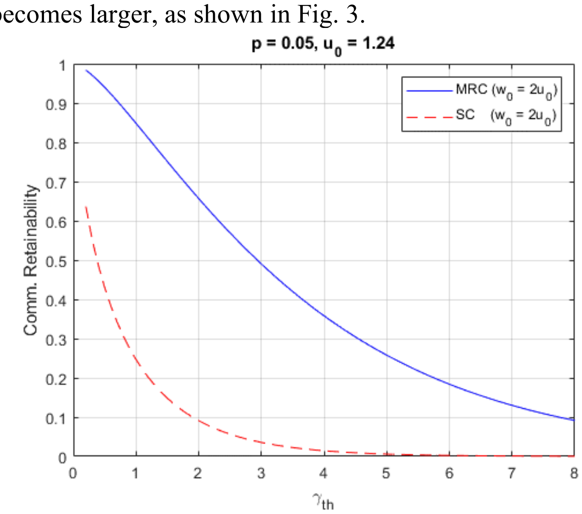

Figure 2. Satellite-ground channel; severe fading; small w_0.

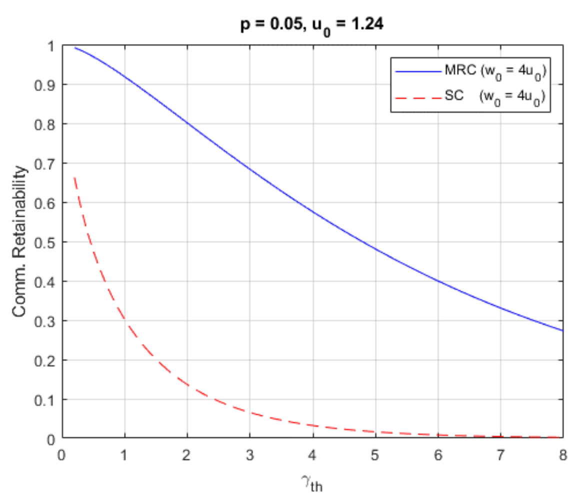

Figure 3. Satellite-ground channel; severe fading; large w_0.

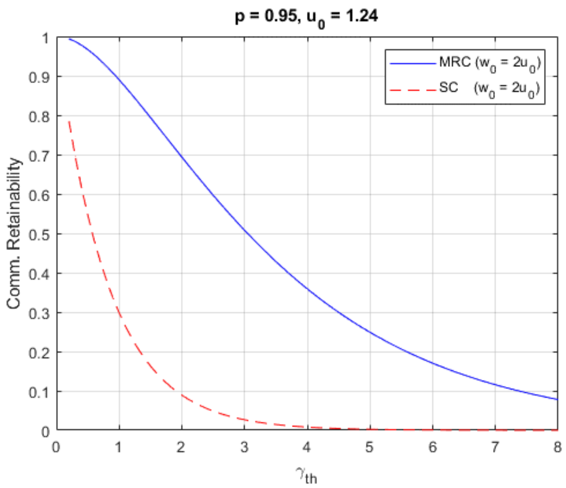

Figure 4. Satellite-ground channel; moderate fading; small w_0.

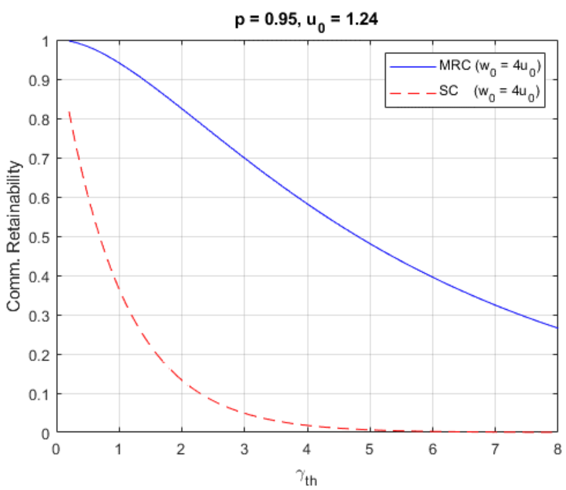

Figure 5. Satellite-ground channel; moderate fading; large w_0.

However, for the same ratio of w_0 to u_0, the effects of fading become insignificant. This is observed between Fig. 2 and Fig. 4, or between Fig. 3 and Fig. 5. This is mainly due to the relatively large propagation path loss, compared with the small-scale fading loss. Overall, from these profiles, it seems that MRC performs much better than SC. Since the adopted parameters are representative, the MRC scheme should be adopted by the receivers.

VI. Conclusion

In the next wave of evolution of wireless communication networking, some compact devices in IoRT are expected to get access from LEO satellites. In this paper, we evaluate the communication retainability of a generic system that consists of one branch from the LEO satellite and the other from the UAV. The fading model is based on the composite Hoyt distributions. In the literature, the existing expressions for the probability distribution functions of Hoyt-MRC were excessively complicated, where either some integrals were included, or nested infinite series were involved. Our compact solutions are in terms of the well-known special functions, not involving integrals nor infinite series. Based on the obtained formulas, the MRC scheme is then compared with the SC scheme. Although SC has the least complexity in installation, its communication retainability seems to be much lower than MRC in the Hoyt-Rayleigh composite fading. Therefore, MRC should be adopted.

Appendix

- Derivation of eq. (9)

First, we rewrite the concerned term as:

$$J_z = \int_0^z \exp\left\{-\left[\frac{(1+p^2)^2}{4p^2u_0} - \frac{1}{w_0}\right]u\right\} I_0\left[\frac{(1-p^4)u}{4p^2u_0}\right]du$$

$$= \int_0^z \exp(-\alpha u) I_0(\beta u)du,$$

where

$$\alpha = \frac{(1+p^2)^2}{4p^2u_0} - \frac{1}{w_0} = \frac{(1+p^2)^2 w_0 - 4p^2u_0}{4p^2u_0w_0},$$

$$\beta = \frac{1-p^4}{4p^2u_0}.$$

Note that $\beta \geq 0$ since the Hoyt factor $p \leq 1$. Suppose

$$\frac{w_0}{u_0} > \frac{2}{1+p^2},$$

which leads to $\alpha > \beta$, or $\alpha > 0$. Note that the above inequality is always held for practical scenarios based on those representative parameters shown in Table I.

$$J_z = \frac{1}{\alpha}(H_0 - H),$$

where

$$H_0 = \int_0^\infty \exp(-y) I_0\left(\frac{\beta y}{\alpha}\right)dy,$$

$$H = \int_{\alpha z}^\infty \exp(-y) I_0\left(\frac{\beta y}{\alpha}\right)dy.$$

Note that $\alpha > \beta$ is a sufficient condition for solving H_0, while $\alpha > 0$ is a sufficient condition for solving H. The integral of H_0 can be solved with the aid of a formula found in [18, p.195, eq. (4.16.1)] or [19, eq. (6.611.4)] :

$$H_0 = \frac{w_0(1+p^2)^2 - 4p^2u_0}{2p\sqrt{[w_0(1+p^2) - 2p^2u_0][w_0(1+p^2) - 2u_0]}}.$$

However, the solution for H_1 is more involved. We need to introduce two intermediary variables a and b, such that

$$\begin{cases} \dfrac{a^2+b^2}{2} = \alpha z, \\ \dfrac{2ab}{a^2+b^2} = \dfrac{\beta}{\alpha}. \end{cases}$$

Solving the variables a and b, we obtain

$$a^2 = h_1 \pm h_2,\ b^2 = h_1 \mp h_2,$$

where

$$\begin{cases} h_1 = \alpha z = \left(\dfrac{(1+p^2)^2}{4p^2 u_0} - \dfrac{1}{w_0} \right) z, \\ h_2 = z\sqrt{\left(\dfrac{1+p^2}{2p^2 u_0} - \dfrac{1}{w_0} \right)\left(\dfrac{1+p^2}{2u_0} - \dfrac{1}{w_0} \right)}. \end{cases}$$

A key relation for the ongoing derivation is:

$$ab = \frac{(1-p^4)z}{4p^2 u_0}.$$

Note that the values of a and b are symmetrical. Without loss of generality, we choose $b > a > 0$. Accordingly,

$$a^2 = h_1 - h_2,\ b^2 = h_1 + h_2.$$

Next, based on the results in [20, p. 586, eqs. (A-3-1) and (A-3-3)], we derive:

$$H = \frac{(1+p^2)^2 w_0 - 4p^2 u_0}{2p\sqrt{[(1+p^2)w_0 - 2p^2 u_0][(1+p^2)w_0 - 2u_0]}} \times \left\{ 2Q(a,b) - \exp\left(-\frac{a^2+b^2}{2} \right) I_0(ab) \right\}$$
$$= H_0 \left\{ 2Q(a,b) - \exp\left(-\frac{a^2+b^2}{2} \right) I_0(ab) \right\}.$$

Therefore,

$$H_0 - H$$
$$= H_0 \left\{ 1 - 2Q(a,b) + \exp\left(-\frac{a^2+b^2}{2} \right) I_0(ab) \right\}$$
$$= \frac{w_0(1+p^2)^2 - 4p^2 u_0}{2p\sqrt{[w_0(1+p^2) - 2p^2 u_0][w_0(1+p^2) - 2u_0]}} \times \left\{ 1 - 2Q(a,b) + \exp\left(-\frac{a^2+b^2}{2} \right) I_0(ab) \right\}.$$

Finally, we have

$$J_z = \frac{2pu_0 w_0}{\sqrt{[w_0(1+p^2) - 2p^2 u_0][w_0(1+p^2) - 2u_0]}} \times \left\{ 1 - 2Q(a,b) + \exp\left(-\frac{a^2+b^2}{2} \right) I_0(ab) \right\}$$
$$= \frac{2pu_0 w_0}{\sqrt{[w_0(1+p^2) - 2p^2 u_0][w_0(1+p^2) - 2u_0]}} \times \left\{ 1 - 2Q(a,b) + \exp\left(\frac{z}{w_0} - \frac{(1+p^2)^2 z}{4p^2 u_0} \right) I_0\left[\frac{(1-p^4)z}{4p^2 u_0} \right] \right\}.$$

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